Favour Akpavanta
19/ENG06/1005
Mechanical Engineering

Integrate the following with respect to their variables

1.) $x^{1/2} \ln x$

$u = \ln x \qquad dv = x^{1/2}$

$du/dx = 1/x \qquad v = \frac{x^{3/2}}{3/2}$

$\int u\,dv = uv - \int v\,du$

$= \ln x \cdot \frac{x^{3/2}}{3/2} - \int \frac{x^{3/2}}{3/2} \cdot \frac{1}{x}$

$= \frac{x^{3/2}}{3/2} \ln x - \frac{1}{3/2} \int \frac{x^{3/2}}{x}$

$= \frac{x^{3/2}}{3/2} \ln x - \frac{1}{3/2} \cdot \frac{x^{3/2}}{3/2}$

$= \frac{x^{1.5}}{1.5} \ln x - \frac{x^{3/2}}{9/4} + C$

2.) $2 \cos 6t \cos t$

$2 \int \cos(6t) \cos t$

$u = 6t, \quad du = 2$

$2 \int \cos 6t \cos t\,dt$

$u = 6t, \quad du = 2 \cos t\,dt$ so $1/2\,du = \frac{2 \cos t\,dt}{2} = \cos t\,dt$

$= 2 \int \cos u \cos t$

$2 \int \cos u \frac{1}{2} du$

$2 \times 1/2 \int \cos u\,du$

Substituting $u$

$\int (\sin 6t) + C$

3.) $\sin^3 x + \cos^4 x$

$\sin x (\sin^2 x \cos^4 x)$

Since $\sin^2 x + \cos^2 x = 1$

$\sin x (1 - \cos^2 x) \cos^4 x \, dx$

$\sin x (\cos^4 x - \cos^6 x) \, dx$

$du = \sin dx$ and $dx = \frac{du}{\sin x}$

$= \sin x (u^4 - u^6) \cdot \frac{du}{\sin x}$

$\int u^4 - u^6 \, du$

$x^n \, n dx = \frac{x^{n+1}}{n+1} + C$

$= \frac{1}{4} u^5 - \frac{1}{6} u^7 + C$

$= \frac{1}{4} (\cos x)^5 - \frac{1}{6} (\cos x)^7 + C$

$= \frac{1}{4} \cos^5 x - \frac{1}{6} \cos^7 x + C$