30/04/2020 Dr OYELAMI'S Group
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19/ENG05/010 Mechatronics Engineering

1) $\lim_{x\to 0}\left[\frac{4x^2-\sin x}{x^3}\right] = \lim_{x\to 0}\left[\frac{8x-\cos x}{3x^2}\right]$

$= \lim_{x\to 0}\left[\frac{8+\sin x}{6x}\right] = \lim_{x\to 0}\left[\frac{\cos x}{6}\right]$

$= \lim_{x\to \infty}\left[\frac{\cos 0}{6}\right] = \frac{1}{6}$

2) $y = \frac{7x^2\cos 8x}{e^{3x}}$

$u = 7x^2 \quad v = \cos 8x \quad w = e^{3x}$

$\frac{du}{dx} = 14x \quad \frac{dv}{dx} = -8\sin 8x \quad \frac{dw}{dx} = 3e^{3x}$

$\frac{dy}{dx} = y\left[\frac{1}{u}\cdot\frac{du}{dx} + \frac{1}{v}\cdot\frac{dv}{dx} - \frac{1}{w}\cdot\frac{dw}{dx}\right]$

$\frac{dy}{dx} = y\left[\frac{1}{7x^2}\cdot 14x + \frac{1}{\cos 8x}\cdot -8\sin 8x - \frac{1}{e^{3x}}\cdot 3e^{3x}\right]$

$\frac{dy}{dx} = y\left[\frac{2}{x} - 8\tan 8x - 3\right]$

$\frac{dy}{dx} = \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{2}{x} - 8\tan 8x - 3\right]$

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3) $y = \cos 5x^2 + 6x$

let $v = 5x^2 + 6x$

$\frac{dv}{dx} = 10x + 6$

$y = \cos v$

$\frac{dy}{dv} = -\sin v$

$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx}$

$\frac{dy}{dx} = -\sin v \times 10x + 6$

$\frac{dy}{dx} = -(10x + 6)\sin v$

$\frac{dy}{dx} = -(10x + 6)\sin(5x^2 + 6x)$

4) a) $\int \frac{3\,dx}{4x + 1}$

let $v = 4x + 1$

$dv = 4\,dx$

$dx = \frac{dv}{4}$

$= \int \frac{3\frac{dv}{4}}{v}$

$= \frac{3}{4} \ln v$

$= \frac{3}{4} \ln(4x + 1)$

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b) $\int \frac{dx}{x^2+49} = \int \frac{dx}{x^2+7^2}$

let $x = 7\tan\theta$; $\theta = \tan^{-1}\frac{x}{7}$

$dx = 7\sec^2\theta\, d\theta$

$7^2+x^2 = 7^2 + 7^2\tan\theta$

$= 7^2(1+\tan^2\theta)$

recall $1+\tan^2\theta = \sec^2\theta$ (trigonometrical)

$7^2+x^2 = 7^2\sec^2\theta$

$\int \frac{7\sec^2\theta\, d\theta}{7^2\sec^2\theta} = \frac{1}{7}\int d\theta$

$= \frac{1}{7}[\theta] + C$

recall $\theta = \tan^{-1}\frac{x}{7}$

$= \frac{1}{7}\tan^{-1}\frac{x}{7} + C$

e) $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\, dx$

$= \left[\frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x\right] + C$

$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$

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(d) $\int x\sqrt{9+x^2} = \int x\sqrt{u} \cdot \frac{du}{2x}$

let $u = 9 + x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\therefore \int x\sqrt{9+x^2} = \int x\sqrt{u} \cdot \frac{du}{2x}$

$= \frac{1}{2}\int u^{1/2} \cdot du$

$= \frac{1}{2}\left[\frac{2u^{3/2}}{3}\right] + C$

$= \frac{u^{3/2}}{3} + C$

$= \frac{1}{3}\sqrt{(9+x)^3} + C$

BCS