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MECHATRONICS ENGINEERING
19/ENG05/043

1) $\lim_{x\to 0}\left[\frac{4x^2-\sin x}{x^3}\right]$

Using direct substitution : $\lim\left[\frac{4(0)^2-\sin(0)}{(0)^3}\right] = 0-0$
$= 0$ (undefined)

Using L'Hopital's rule : $\lim_{x\to 0}\left(\frac{4x^2-\sin x}{x^3}\right) = \lim_{x\to 0}\left[\frac{8x-\cos x}{3x^2}\right]$

$= \left[\frac{8+\sin x}{6x}\right]$ $\therefore \lim_{x\to 0}\left[\frac{8+\sin x}{6x}\right] = \lim_{x\to 0}\left[\frac{\cos x}{6}\right] = \frac{\cos(0)}{6} = 1/6$

$\lim_{x\to 0}\left[\frac{4x^2-\sin x}{x^3}\right] = 1/6$

2) $y = \left(\frac{7x^2\cos 8x}{e^{3x}}\right)$

Let $u = 7x^2$, $v = \cos 8x$, $w = e^{3x}$

$\frac{du}{dx} = 14x$, $\frac{dv}{dx} = -8\sin 8x$, $\frac{dw}{dx} = 3e^{3x}$

$dy/dx = y\left[1/7x^2(14x) + 1/\cos 8x(-8\sin 8x) + 1/e^{3x}(3e^{3x})\right)$

$dy/dx = y\left(2/x - 8\cot 8x + 3\right)$

$dy/dx = \left(\frac{7x^2\cos 8x}{e^{3x}}\right)\left[2/x - 8\cot 8x + 3\right]$

3) $y = \cos(5x^2+6x)$

$u = 5x^2+6x$, $y = \cos u$

$\frac{du}{dx} = 10x+6$, $\frac{dy}{du} = -\sin u$

$dy/dx = du/dx \times dy/du$

$= 10x+6 \times (-\sin u)$

$dy/dx = -10x + 6\sin u$

Since $u = 5x^2 + 6x$.

$dy/dx = -10x + 6\sin(5x^2 + 6x)$

4) a) $\int \frac{3}{4x+1}\,dx = 3\int \frac{dx}{4x+1}$

let $u = 4x+1$

$du/dx = 4$

$\therefore dx = \cancel{4/du}\ du/4$

$3\int \frac{dx}{4x+1} = 3\int \frac{du}{4u} = 3/4 \int \frac{du}{u} = 3/4 \int 1/u\ du$

$= 3/4 \ln u + c$.

$\therefore \int \frac{3}{4x+1}\,dx = 3/4 \ln(4x+1) + C$

b) $\int \frac{dx}{(x^2+49)}$

let $x = a\tan\theta$, where $a = 7$

$x = 7\tan\theta$

$\frac{dx}{d\theta} = 7\sec^2\theta$

$dx = 7\sec^2\theta\, d\theta$

$x^2 + 49 = \cancel{7\tan}(7\tan\theta)^2 + 49$

$= 7^2\tan^2\theta + 49$

$49\tan^2\theta + 49$

$\int \frac{dx}{(x^2+49)} = \int \frac{7\sec^2\theta\, d\theta}{49\tan^2\theta + 49} = \int \frac{7\sec^2\theta}{49(\tan^2\theta + 1)}\, d\theta = 1/7 \int \frac{\sec^2\theta}{\tan^2\theta + 1}$

$\int \frac{dx}{(x^2+49)} = 1/7 \int \frac{\sec^2\theta\, d\theta}{\sec^2\theta} = 1/7 \int d\theta = \cancel{1/7 + c}\ 1/7\,[\theta] + C$

$$\int \frac{dx}{(x^2+49)} = \frac{1}{7}\tan^{-1}\frac{x}{7} + C$$

c) $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\, dx$

$$= \left[\frac{1}{6}e^{6x} + \frac{9x^4}{9} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x\right] + c$$

$$\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\, dx = \cancel{\frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C}$$

$$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$$

d) $\int x\sqrt{9+x^2}\, dx$

let $u = 9+x^2$

$$\frac{du}{dx} = 2x \quad \therefore dx = \frac{du}{2x}$$

$$\int x\sqrt{9+x^2}\, dx = \int x \times \sqrt{u} \times \cancel{du/dx}\; du/2x$$

$$= \frac{1}{2}\int \frac{\sqrt{u}}{1}\, du$$

$$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right] + C = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}u^{3/2}}{3}\right] + c$$

$$= \frac{1}{3}\left[u^{3/2}\right] + C$$

$$= \frac{1}{3}(9+x^2)^{3/2} + C$$

$$\therefore \int x\sqrt{9+x^2}\, dx = \frac{1}{3}(9+x^2)^{3/2} + C$$