

- 1,  $x^{-1} \ln x$
- 2,  $2 \cos^2 t \cos t$
- 3,  $\sin^3 x \cos^2 x$

Solution

1,  $x^{1/2} \ln x$   
 $\int x^{1/2} \ln x$

$u = x^{1/2} \quad du = \frac{1}{2} x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2} x^{-1/2}$

$\int u \frac{du}{dx} + \int u \frac{du}{dx}$

$\ln x \int \frac{dx}{2x^{3/2}} + x^{1/2} \int \frac{d \ln x}{dx}$

$\ln x \left[ \frac{2x^{-3/2}}{-3/2} \right] + x^{1/2} \left[ \frac{1}{2x} \right] + C$

$\frac{2 \ln x \cdot (-2) x^{-3/2}}{3} + \frac{x^{1/2}}{2x} + C$

$\frac{2x^{3/2} \ln x}{3} + \frac{\sqrt{x}}{2} + C$

$$2) \int 2 \cos bt \cos t dt = 12 \int \cos bt \cos t dt$$

$$\begin{aligned} \text{let } A &= bt \quad B = t \\ \cos A \cos B &= \frac{1}{2} [\cos(A+B) + \cos(A-B)] \\ &= \frac{1}{2} [\cos(b+t) + \cos(b-t)] \\ &= \frac{1}{2} [\cos 7t + \cos 5t] \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \int 2 \cos bt \cos t dt &= \frac{1}{2} \int (2 \cos 7t + \cos 5t) dt \\ &= \frac{1}{2} \left[ \frac{2 \sin 7t}{7} - \frac{\sin 5t}{5} \right] \end{aligned}$$

$$= \frac{1}{7} \sin 7t - \frac{1}{10} \sin 5t + C$$

$$3) \int \sin^3 x \cos^4 x = \int \sin^m x \cos^n x$$

Since  $m$  is odd

$$u = (\cos^2 x)^2$$

$$du/dx = -\sin x \Rightarrow dx = -du/\sin x$$

$$3) \int \sin^3 x \cos^4 x dx$$

$$\int \sin^3 x (\cos^2 x)^2 dx$$

$$u = (\cos^2 x)^2$$

$$du/dx$$

$$2) \int 2 \cos 6t \cos t dt = 12 \int \cos 6t \cos t dt$$

$$\begin{aligned} \text{Let } A &= 6t \quad B = t \\ \cos A \cos B &= \frac{1}{2} [\cos(A+B) + \cos(A-B)] \\ &= \frac{1}{2} [\cos(6+t) + \cos(6-t)] \\ &= \frac{1}{2} [\cos 7t + \cos 5t] \\ &= \frac{1}{2} \end{aligned}$$

$$\int 2 \cos 6t \cos t dt = \frac{1}{2} \int (2 \cos 7t + \cos 5t) dt$$

$$= \frac{1}{2} \left[ \frac{2 \sin 7t}{7} - \frac{\sin 5t}{5} \right]$$

$$= \frac{2 \sin 7t}{7} - \frac{\sin 5t}{5} + C$$

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$$\int \sin^3 x (\cos^2 x)^2 dx$$

$$u = (\cos^2 x)^2$$

$$du/dx$$