Matlon

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① $A = 3i + 7j - 2k$

$B = i + 2j + 7k$

$C = 9i - 4j + 6k$

Angle between A and C

$\cos\theta = ll' + mm' + nn'$

Direct cosines of A

$A = |A| = \sqrt{(3)^2 + (7)^2 + (-2)^2}$

$= \sqrt{62}$

$l = \frac{3}{\sqrt{62}}, \quad m = \frac{7}{\sqrt{62}}, \quad n = \frac{-2}{\sqrt{62}}$

$\therefore (l, m, n) = \left[\frac{3}{\sqrt{62}}, \frac{7}{\sqrt{62}}, \frac{-2}{\sqrt{62}}\right)$

Direct cosines of C

$C = |C| = \sqrt{(9)^2 + (-4)^2 + (6)^2}$

$= \sqrt{81 + 16 + 36}$

$= \sqrt{133}$

$l' = \frac{9}{\sqrt{133}}, \quad m' = \frac{-4}{\sqrt{133}}, \quad n' = \frac{6}{\sqrt{133}}$

$\therefore (l', m', n') = \left[\frac{9}{\sqrt{133}}, \frac{-4}{\sqrt{133}}, \frac{6}{\sqrt{133}}\right)$

$\cos\theta = \frac{3}{\sqrt{62}} \times \frac{9}{\sqrt{133}} + \frac{7}{\sqrt{62}} \times \frac{-4}{\sqrt{133}} + \frac{-2}{\sqrt{62}} \times \frac{6}{\sqrt{133}}$

$= 0.2973 + (-0.3083) + (-0.1721)$

$\cos\theta = -0.1831$

$\theta = 93.23^\circ$

(10) Angle between B and C

$\cos\theta = ll' + mm' + nn'$

$AB = |BA| = \sqrt{(4)^2 + (3)^2 + (7)^2}$

$= \sqrt{1 + a + 4 + 4}$

$= \sqrt{54}$

$l = \frac{1}{\sqrt{54}}, m = \frac{3}{\sqrt{54}}, n = \frac{7}{\sqrt{54}}$

$\therefore C(l, m, n) = \left[\frac{1}{\sqrt{54}}, \frac{3}{\sqrt{54}}, \frac{7}{\sqrt{54}}\right.$

$c = |CI| = \sqrt{133}$

$(l', m', n') = \left(\frac{9}{\sqrt{133}}, \frac{-4}{\sqrt{133}}, \frac{6}{\sqrt{133}}\right)$

$\cos\theta = \frac{1}{\sqrt{54}} \times \frac{9}{\sqrt{133}} + \frac{3}{\sqrt{54}} \times \frac{-4}{\sqrt{133}} + \frac{7}{\sqrt{54}} \times \frac{6}{\sqrt{133}}$

$= 0.1016 + (-0.1355) + (0.4741)$

$= 0.4403$

$\theta = \cos^{-1} 0.4403$

$= 63.88°$

(11) $T(t) = \frac{r(t)}{|r'(t)|}$

$A + B + C = 13i + 6j + 11k$

$r(t) = (13, 6, 11)$

$\vec{T}(t) = \frac{\vec{r(t)}}{|r(t)|} = \frac{(13, 6, 11)}{\sqrt{(13)^2 + (6)^2 + 11^2}}$

$= \frac{(13, 6, 11)}{\sqrt{326}}$

$= \left(\frac{13}{\sqrt{326}}, \frac{6}{\sqrt{326}}, \frac{11}{\sqrt{326}}\right)$

(a) $r(t) = \langle -8t^2, t^2 - 4t, t+1 \rangle$

$r'(t) = \langle -16t, 2t-4, 1 \rangle$

$|r'(t)| = \sqrt{(-16t)^2 + (2t-4)^2 + 1^2}$

$= \sqrt{256t^2 + 4t^2 - 16t + 16 + 1}$

$= \sqrt{260t^2 - 16t + 17}$

let $t = 1$

$= \sqrt{260(1)^2 - 16(1) + 17}$

$= \sqrt{260 - 16 + 17}$

$= \sqrt{261}$

$= 3\sqrt{29}$

$= 16.1554 \approx 16.16$

(b) ~~$(A \times B) \times C$~~

(3) (A×B)×C.

$$A\times B = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & -4 \\ -2 & 1 \end{vmatrix} - j\begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k\begin{vmatrix} 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$= i[(2)(1)-(-2)(-4)] - j[(4)(1)-(8)(-4)] + k[(4)(-2)-8(2)]$$

$$= (2-8)i - (4+32)j + (-8-16)k$$

$$= -6i - 36j - 24k$$

$$(A\times B)\times C = \begin{vmatrix} i & j & k \\ -6 & -36 & -24 \\ 1 & -3 & -3 \end{vmatrix}$$

$$= i\begin{vmatrix} -36 & -24 \\ -3 & -3 \end{vmatrix} - j\begin{vmatrix} -6 & -24 \\ 1 & -3 \end{vmatrix} + k\begin{vmatrix} -6 & -36 \\ 1 & -3 \end{vmatrix}$$

$$= i[(36)(-3)-(-24)(-3)] - j[(6)(-3)-(1)(-24)] + k[(6)(-3)-(-36)(1)]$$

$$= i[108-(-46)] - j[18-(-24)] + k[-24-36]$$

$$= 204i + 42j - 60k$$

$$= 204i \pm 42j - 60k$$