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MAT 102

1 $\int x^2 \sin x \, dx$

Solu.

$\int u\,dv = uv - \int v\,du$ ①

for $\int u\,dv = \int x^2 \sin(x)\,dx$

$u = x^2$, $du/dx = 2x$

$du = 2x\,dx$

$dv = \sin(x)\,dx$

$\int dv = \int \sin(x)\,dx$

$v = -\cos(x)$

$\therefore$

$\int x^2 \sin(x)\,dx = -x^2\cos(x) - \int -2x\cos(x)\,dx$

$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2\int x\cos(x)\,dx$ --②

$v = x \Rightarrow \frac{dv}{dx} = 1 \Rightarrow du = dx$

~~dv~~

$dv = \cos(x)\,dx \Rightarrow \int dv = \int \cos(x)\,dx$

$v = \sin(x)$

$\int x\cos(x)\,dx = x\sin(x) - \int \sin(x)\,dx$

Since $\int \sin(x)\,dx = -\cos(x)$. this becomes

$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$ --- ③

$\therefore \int x^2 \sin(x)\,dx = -x^2\cos(x) + 2\int x\cos(x)\,dx$

Sub in (3) into (2)

$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2(x\sin(x) + \cos(x))$

$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C$

2. $\int 3te^{2t}\,dt$

$\int u\,dv = uv - \int v\,du$

let $u = 3t$, $dy/dt = 3$

$\therefore du = 3dt$

let $dv = e^{2t}\,dt$

$v = 1/2\, e^{2t}$

$\int 3t(e^{2t})dt = 3t(1/2)e^{2t} - \int 1/2 e^{2t} 3dt$

$= 3/2\, te^{2t} - 3/4\, e^{2t}$

3) $\int 2x^2 \times \ln x\,dx$

commutative property

$2x\int x^2 \ln(x)\,dx$

$2x\ln(x) \times x^2\,dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln(x)$

$dv = x^2\,dx$

$v = \int dv \qquad du = 1/x\,dx \qquad v = x^3/3$

$$2\left(\ln x \cdot x^{3/3} - \int x^{3/3} \times \frac{1}{x}\,dx\right)$$

$$\int u\,dv = uv - \int v\,du$$

$$\int 2x^2 \ln(x)\,dx = 2\left(\ln x \cdot x^{3/3} - \int x^{3/3} \times \frac{1}{x}\,dx\right)$$

$$2x^2 \ln x\,dx = 2\left(\ln x \cdot x^{3/3} - \int x^{2/3}\,dx\,dx\right)$$

$$= 2\left(\ln x \times \frac{x^3}{3} - \frac{1}{3} x \int x^2\,dx\right)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int x^2\,dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$$

$$= 2\left(\ln(x) \times \frac{x^3}{3} - \frac{1}{3} \times \frac{x^3}{3}\right)$$

$$= \frac{2x^3 \times \ln(x)}{3} - \frac{9x}{2}$$

$$\int 2x^2 \ln x\,dx = \frac{2x^3 \times \ln x}{3} - \frac{2x^3}{9} + C$$

4 $$\int \frac{2x - 3x^2}{1-x}\,dx = \int \frac{2x}{1-x} - \frac{3x^2}{1-x}\,dx$$

Property of Integral

$$\int f(x) \pm y(x)\,dx = \int f(x)\,dx \pm \int y(x)\,dx$$

$$\int \frac{2x}{1-x}\,dx - \int \frac{3x^2}{1-x}\,dx$$

calculate Indefinite Integral

$$2-2x-2\ln(|1-x|)+\frac{-9+6x+3}{2}+3\ln(|1-x|)$$

Simplify

$$\frac{-3+2x+3x^2}{2}+\ln(|1-x|)$$

$$\therefore \int \frac{2x-3x^2}{1+x} = \frac{-3+2x+3x^2}{2}+\ln(|1-x|)-c$$