Find the limit of the function

$$\lim_{x \to 0} \frac{4x^2 - \sin x}{x^3}$$

Using L'Hop.tal's rule

$$\lim_{x \to 0} \frac{8x - \cos x}{3x^2}$$

$$\lim_{x \to 0} \frac{8 + \sin x}{6x}$$

$$\lim_{x \to 0} \frac{0 - \cos x}{6} = \frac{-\cos 0}{6} = -\frac{1}{6}$$

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1 find the limit of the function

$\frac{4x^2 - \sin x}{x^3}$ as $x \to 0$

$\lim_{x \to 0} \frac{4x^2 - \sin x}{x^3}$

= Direct substitution

$= \frac{4(0)^2 - \sin 0}{0^3} = \frac{0}{0}$

2 If $y = \frac{7x^2 \cos 8x}{e^{3x}}$ find the derivative of y with respect to x

$\frac{dy}{dx} = \frac{uv}{w}\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}\right)$

$u = 7x^2 \quad du/dx = 14x$

$v = \cos 8x \quad dv/dx = -8\sin 8x$

$w = e^{3x} \quad dw/dx = 3e^{3x}$

$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}}\left(\frac{1}{7x^2} \cdot 14x + \frac{1}{\cos 8x} \cdot -8\sin 8x - \frac{1}{e^{3x}} \cdot 3e^{3x}\right.$

$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}}\left(\frac{2}{x} - 8\tan 8x - 3\right)$

3 If $y = \cos(5x^2 + 6x)$ find dy/dx

$y = \cos u$

$u = 5x^2 + 6x$

$du/dx = 10x + 6$

$\frac{dy}{du} = -\sin u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\int \frac{1}{7} \frac{\sec^2\theta}{\sec^2\theta} \, d\theta$$

$$\frac{1}{7} \int d\theta$$

$$\frac{1}{7} [\theta] + C$$

$$\frac{1}{7} \tan^{-1} \frac{x}{7} + C$$

4c $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x) \, dx$

$$\frac{e^{6x}}{6} + \frac{9x^4}{4} - \frac{\cos 7x}{7} - \frac{\sin 8x}{8} + C$$

4d $\int x \sqrt{9+x^2} \, dx$

$x \int u^{1/2} \, du$

$du/dx = 2x$

$dx = 1/2x \, du$

$x \int u^{1/2} \cdot \frac{1}{2x} \, du$

$\frac{1}{2} \int u^{1/2} du$

$\frac{1}{2} \frac{u^{1/2+1}}{1/2+1} + C$

$\frac{1}{2} \frac{u^{3/2}}{3/2} + C$

$\frac{1}{2} \frac{2}{3} u^{3/2} + C$

$\frac{1}{3} (9+x^2)^{3/2} + C$

$$\frac{dy}{dx} = -\sin u \times 10x+6$$

$$\frac{dy}{dx} = 10x+6 \cdot -\sin(5x^2+6x)$$

$$\frac{dy}{dx} = -10x-6\sin(5x^2+6x)$$

4a) find the integral of the following

$$\int \frac{3\,dx}{4x+1}$$

$$3\int \frac{dx}{4x+1}$$

$$3\int \frac{1}{u}\,dx$$

$u = 4x+1 \quad du/dx = 4 \quad dx = 1/4\,du$

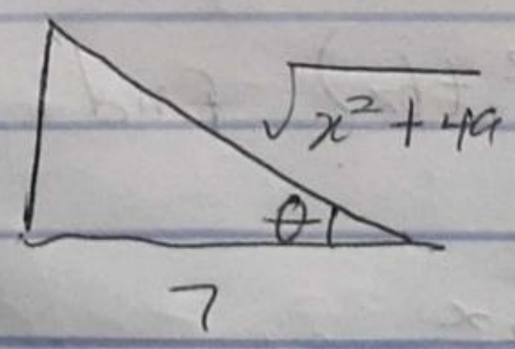
$$3\int \frac{1}{u}\cdot\frac{1}{4}\,du$$

$$\frac{3}{4}\int \frac{1}{u}\,du$$

$$\frac{3}{4}\ln u + C$$

$$\frac{3}{4}\ln(4x+1) + c$$

4b) $$\int \frac{dx}{(x^2+49)}$$



$x = 7\tan\theta \qquad \theta = \tan^{-1}\frac{x}{7}$

$dx = 7\sec^2\theta\,d\theta$

$$\int \frac{7\sec^2\theta\,d\theta}{(7\tan\theta)^2+7^2}$$

$$\int \frac{7\sec^2\theta\,d\theta}{7^2(\tan^2\theta+1)} \qquad \sec^2\theta = \tan^2\theta+1$$