

Integrate

$$\int x^{1/2} \ln x \, dx$$

$$\int x^{1/2} \ln x$$

$$u = \ln x \quad dv = x^{1/2} dx$$

$$du = \frac{1}{x} dx \quad v = \frac{2x^{3/2}}{3}$$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$\int x^{1/2} \ln x = \frac{2x^{3/2}}{3} - \int \frac{2x^{3/2}}{3} \cdot \frac{1}{x} dx$$

$$= \frac{2x^{3/2}}{3} - \int \frac{2x^{1/2}}{3} dx$$

$$\int \frac{2x^{1/2}}{3} dx = \frac{2x^{1/2+1}}{3(1/2+1)} = \frac{2x^{3/2}}{3(3/2)} = \frac{2x^{3/2}}{9/2} = \frac{4x^{3/2}}{9}$$

$$\therefore \int x^{1/2} \ln x \, dx = \frac{2x^{3/2}}{3} \ln x - \frac{4x^{3/2}}{9} + C$$

2i  $2 \cos 6t \cos t$

$$\int 2 \cos 6t \cos t \, dt$$

$$2 \cos A \cos B = 2 \left[ \frac{1}{2} (\cos(A+B) + \cos(A-B)) \right]$$

$$= \cos(A+B) + \cos(A-B)$$

$$\begin{aligned} \int 2 \cos 6t \cos t \, dt &= \int \cos 7t + \cos 5t \\ &= \int \cos 7t \, dt + \int \cos 5t \, dt \\ &= \frac{\sin 7t}{7} + \frac{\sin 5t}{5} + C \end{aligned}$$

$$i) \sin^3 x \cos^4 x$$

$$\int \sin^3 x \cos^4 x \, dx$$

$$\text{Let } u = \cos x$$

$$du/dx = -\sin x \implies dx = -du/\sin x$$

$$\text{Recall: } \cos^2 x + \sin^2 x = 1$$

$$\therefore \sin^2 x = 1 - \cos^2 x$$

$$= \int \sin^3 x \cdot u^4 \cdot \frac{-du}{\sin x} = - \int \sin^2 x \cdot u^4 \cdot du$$

$$= - \int (1 - \cos^2 x) \cdot u^4 \cdot du$$

$$= - \int (1 - u^2) \cdot u^4 \cdot du$$

$$= - \int (u^4 - u^6) \cdot du$$

$$= - \left[ \frac{u^5}{5} - \frac{u^7}{7} \right] + C$$

$$= - \frac{u^5}{5} + \frac{u^7}{7} + C$$

$$\frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$