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1) $\frac{dy}{dx}\left(\frac{4x^2 - \sin x}{x^3}\right)$

$= \frac{4x^2}{x^2 x} - \frac{\sin x}{x^3}$

$= \frac{4}{x} - \frac{\sin(x)}{x^3}$

$= \frac{d}{dx}\left(\frac{4}{x}\right) - \frac{d}{dx}\frac{\sin(x)}{x^3}$

$= \frac{-4}{x^2} - \frac{x\cos x - 3\sin x}{x^4}$

$= -4 \times \frac{1}{x^2} - \frac{\cos(x)(x)^3 - \sin(x) \times 3x^2}{(x^3)^2}$

$= \frac{-4x^2 + x\cos x - 3\sin(x)}{x^4}$

$= \frac{-4x^2 + x\cos x - 3\sin x}{x^4}$

$\lim_{x \to 0} = \frac{-4x^2 + x\cos x - 3\sin x}{x^4}$

$= \cancel{-4x^2}\ 0 + 0 + 0 + 0 = 0$

(2) $\frac{dy}{dx}\left(\frac{7x^2\cos 8x}{e^{3x}}\right)$

using quotient rule

$$\frac{dy}{dx}\left(\frac{u}{v}\right) = \frac{\frac{d}{dx}(u)\times v - u\frac{d}{dx}(v)}{v^2}$$

$$= \frac{\frac{d}{dx}(7x^2 \times \cos 8x)\times e^{3x} - 7x^2\cos 8x \times \frac{d}{dx}(e^{3x})}{(e^{3x})^2}$$

$$= \frac{7\times 2x\cos(8x) + 7x^2\times(-\sin(8x)\times 8)e^{3x} - 7x^2 - \cos(8x)\times\frac{d}{dx}(e^{3x})}{(e^{3x})^2}$$

$$= \frac{7\times 2x\cos(8x + 7x^2(-\sin(8x))\times 8)e^{3x} - 7x^2\cos 8x \times e^{3x}\times 3}{(e^{3x})^2}$$

$$= \frac{7\times 2x\times\cos(8x) + 7x^2\times(-\sin(8x)\times 8)e^{3x} - 7x^2\times\cos 8x \times e^{3x}\times 3}{(e^{3x})^2}$$

$$= \frac{14x\times\cos 8x - 56x^2\times\sin 8x - 21x^2\times\cos 8x}{e^{3x}}$$

$$= \frac{14x\cos 8x - 56x^2\sin 8x - 21x^2\cos 8x}{e^{3x}}$$

$$= \frac{14x(\cos 8x) - 56x^2(\sin 8x) - 21x^2\cos 8x + C, C\in R}{e^{3x}}$$

(3)

(3) $\frac{dy}{dx}(\cos(5x^2+6x))$

$= \frac{d}{dx}(v(u)) = \frac{d}{du} v(u) \times \frac{d}{dx}(u)$ (using chain rule)

$= \frac{d}{dx}(\cos(y))' \times \frac{d}{dx}(5x^2+6x$

$= -\sin(y) \times \frac{d}{dx}(5x^2+6x)$

$= -\sin(y) \times (5\times 2x+6)$

$= -\sin(5x^2+6x) \times (5\times 2x+6)$

$= -\sin(5x^2+6x) \times (10x+6)$

$= -\sin(5x^2+6x)(10x+6$

(4a) $\frac{3dx}{4x+1}$

$= \int \frac{3}{4x+1} dx$

Let $u = 4x+1$

$= \int \frac{3}{4u} du$

$= \frac{3}{4} \times \int \frac{1}{u} du$

$= \frac{3}{4} \times \ln|u|$

$= \frac{3}{4} \times \ln(4x+1)$

$= \frac{3}{4} \times \ln(4x+1) + C, C \in R.$

(4b) $\int \frac{dx}{x^2+49}$

using $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$= \frac{1}{7} \times \tan^{-1}\left(\frac{x}{7}\right)$

$= \frac{\tan^{-1}\left(\frac{x}{7}\right)}{7}$

$= \frac{\tan^{-1}\left(\frac{x}{7}\right)}{7} + C, C \in R.$

(4c) $(e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

$= \int e^{6x}\,dx - \int 9x^3\,dx - \int \sin(7x)\,dx + \int \cos 8x\,dx$

$= \frac{e^{6x}}{6} - \int 9x^3\,dx - \int \sin(7x)\,dx + \int \cos(8x)\,dx$

$= \frac{e^{6x}}{6} - \frac{9x^4}{4} - \int \sin(7x)\,dx + \int \cos 8x\,dx$

$= \frac{e^{6x}}{6} - \frac{9x^4}{4} + \frac{\cos(7x)}{7} + \int \cos(8x)\,dx$

$= \frac{e^{6x}}{6} - \frac{9x^4}{4} + \cos(7x) + \frac{\sin(8x)}{8}$

$= \frac{e^{6x}}{6} - \frac{9x^4}{4} + \frac{\cos(7x)}{7} + \frac{\sin(8x)}{8} + C, C \in R.$

4d $\left(x \times \sqrt{a+x^2}\right) dx$

$x(a+x^2)^{\frac{1}{2}}$

$= \int x\sqrt{a+x^2}\, dx$

let $u = a+x^2$

$= \int \frac{1}{2}\sqrt{u}\, du$

$= \frac{1}{2} \times \int \sqrt{u}\, du$

$= \frac{1}{2} \times \int u^{\frac{1}{2}}$

$= \frac{1}{2} \times \frac{2u\sqrt{u}}{3}$

$= \frac{1}{2} \times \frac{2(a+x^2)\sqrt{a+x^2}}{3}$

$= \frac{(a+x^2)\sqrt{a+x^2}}{3}$

$= \frac{(a+x^2)\sqrt{a+x^2}}{3} + c, \; c \in R.$