(1) $x^{1/2}\ln x\,dx$

$\int x^{1/2}\ln x$

$u = \ln x \qquad dv = x^{1/2}\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{2x^{3/2}}{3}$

$\int u\cdot dv = uv - \int v\cdot du$

$\int x^{1/2}\ln x = \frac{2x^{3/2}}{3} - \int \frac{2x^{3/2}}{3}\cdot\frac{1}{x}\,dx$

$= \frac{2x^{3/2}}{3} - \int \frac{2x^{1/2}\,dx}{3}$

$\int \frac{2x^{1/2}\,dx}{3} = \frac{2x^{1/2+1}}{3(1/2+1)} = \frac{2x^{3/2}}{3(3/2)} = \frac{2x^{3/2}}{9/2} = \frac{4x^{3/2}}{9}$

$\therefore \int x^{1/2}\ln x\,dx = \frac{2x^{3/2}\ln x}{3} - \frac{4x^{3/2}}{9} + C$

2. $2\cos 6t\cos t$

$\int 2\cos 6t\cos t\,dt$

$2\cos A\cos B = \cancel{2}\left[\frac{1}{\cancel{2}}\left[\cos(A+B) + \cos(A-B)\right]\right]$

$= \cos(A+B) + \cos(A-B)$

$\int 2\cos 6t\cos t\,dt = \int \cos 7t + \cos 5t$

$= \int \cos 7t\,dt + \int \cos 5t\,dt$

$= \frac{+\sin 7t}{7} + \frac{\sin 5t}{5} + C$

3i) $\sin^3 x \cos^4 x$

$\int \sin^3 x \cos^4 x \, dx$

Let $u = \cos x$

$du/dx = -\sin x \implies dx - du/\sin x$

Recall: $\cos^2 x + \sin^2 x = 1$

$\therefore \sin^2 x = 1 - \cos^2 x$

$= \int \sin^3 x \cdot u^4 \cdot \frac{-du}{\sin x} = -\int \sin^2 x - u^4 \cdot du$

$= -\int (1 - \cos^2 x) \cdot u^4 \cdot du$

$= -\int (1 - u^2) \cdot u^4 \cdot du$

$= -\int (u^4 - u^6) \cdot du$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$

$= \frac{-u^5}{5} + \frac{u^7}{7} + C$

$\frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$