

1.) Find the limit of the function: $(4x^2 - \sin x) / x^3$ as $x \rightarrow 0$

$$\therefore \lim_{x \rightarrow 0} \left[\frac{4x^2 - \sin x}{x^3} \right]$$

By direct substitution: $\lim_{x \rightarrow 0} \left[\frac{4(0)^2 - \sin(0)}{(0)^3} \right] = \frac{0 - 0}{0}$ undefined

Using L'Hopital's rule:

$$\lim_{x \rightarrow 0} \left[\frac{4x^2 - \sin x}{x^3} \right] = \lim_{x \rightarrow 0} \left[\frac{8x - \cos x}{3x^2} \right] = \left[\frac{8 + \sin x}{6x} \right]$$

$$\lim_{x \rightarrow 0} \left[\frac{8 + \sin x}{6x} \right] = \lim_{x \rightarrow 0} \left[\frac{\cos x}{6} \right] = \frac{\cos(0)}{6} = \frac{1}{6}$$

$$\therefore \lim_{x \rightarrow 0} \left[\frac{4x^2 - \sin x}{x^3} \right] = \frac{1}{6}$$

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2. Find $\frac{dy}{dx}$; If $y = \frac{7x^2 \cos 8x}{e^{3x}}$

Let $U = 7x^2$, $V = \cos 8x$, $W = e^{3x}$

$$\frac{du}{dx} = 14x, \quad \frac{dv}{dx} = -8\sin 8x, \quad \frac{dw}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = y \left[\frac{1}{U} (14x) + \frac{1}{V} (-8\sin 8x) + \frac{1}{W} (3e^{3x}) \right]$$

$$\frac{dy}{dx} = y \left(\frac{2}{x} - 8\cot 8x + 3 \right)$$

$$\frac{dy}{dx} = \left(\frac{7x^2 \cos 8x}{e^{3x}} \right) \left(\frac{2}{x} - 8\cot 8x + 3 \right)$$

3. If $y = \cos(5x^2 + 6x)$ find $\frac{dy}{dx}$

$$U = 5x^2 + 6x \quad y = \cos U$$

$$\frac{du}{dx} = 10x + 6 \quad \frac{dy}{du} = -\sin U$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= 10x + 6 \times -\sin U$$

$$\frac{dy}{dx} = -10x + 6 \sin U$$

$$\text{Since } U = 5x^2 + 6x$$

$$\frac{dy}{dx} = -10x + 6 \sin(5x^2 + 6x)$$

4a. Find the integral of: $\int \frac{3}{4x+1} dx$

$$\int \frac{3}{4x+1} dx = 3 \int \frac{dx}{4x+1}$$

$$\text{Let } U = 4x + 1$$

$$\frac{du}{dx} = 4$$

$$\therefore dx = \frac{du}{4}$$

$$3 \int \frac{dx}{4x+1} = 3 \int \frac{1}{4} \frac{du}{4}$$

$$\frac{3}{4} \int \frac{1}{4} du$$

$$= \frac{3}{4} \ln U + C$$

$$\therefore \int \frac{3}{4x+1} dx = \frac{3}{4} \ln(4x+1) + C$$

$$b.) \int \frac{dx}{x^2 + 49}$$

Let $x = a \tan \theta$ where $a = 7$

$$x = 7 \tan \theta$$

$$dx = 7 \sec^2 \theta d\theta$$

$$d\theta$$

$$dx = 7 \sec^2 \theta d\theta$$

$$\therefore x^2 + 49 = (7 \tan \theta)^2 + 49$$

$$= 49 \tan^2 \theta + 49$$

$$= 49 \tan^2 \theta + 49$$

$$= 49(\tan^2 \theta + 1)$$

$$49 + x^2 = 49(1 + \tan^2 \theta)$$

$$\text{Recall: } 1 + \tan^2 \theta = \sec^2 \theta$$

$$49 + x^2 = 49 \sec^2 \theta$$

$$\int \frac{dx}{x^2 + 49} = \int \frac{7 \sec^2 \theta d\theta}{49 \sec^2 \theta}$$

$$= \frac{1}{7} \int d\theta$$

$$= \frac{1}{7} \int 0 \, d\theta + C$$

$$\therefore \int \frac{dx}{x^2 + 49} = \frac{1}{7} \tan^{-1} \frac{x}{7} + C$$

$$c.) \int (e^{6x} + 9x^3 - \sin 7x + \cos 8x) dx$$

$$= \left[e^{6x} + \frac{9x^4}{4} + (\cos 7x + \cos 8x) \right]_{6}^{4}$$

$$= \underline{\underline{e^{6x}}} + \underline{\underline{9x^4}} + \underline{\underline{1}} \cos 7x + \underline{\underline{1}} \cos 8x + C$$

$$d.) \int x \sqrt{9+x^2} dx$$

$$\text{let } u = 9 + x^2$$

$$du = 2x$$

$$dx$$

$$\therefore dx = \frac{du}{2x}$$

$$= \frac{1}{2} \int \sqrt{u} du$$

$$= \frac{1}{2} \left[\frac{u^{\frac{3}{2}+1}}{\frac{3}{2}} \right] + C$$

$$= \frac{1}{2} \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right] + C$$

$$= \frac{1}{3} \left[2u^{\frac{3}{2}} \right] + C$$

$$= \frac{1}{3} [u^{\frac{3}{2}}] + C$$

$$= \frac{1}{3} (9 + x^2)^{\frac{3}{2}} + C$$

$$\therefore \int x \sqrt{9+x^2} dx = \frac{1}{3} (\sqrt{9+x^2})^3 + C$$

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