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MECHATRONICS ENGINEERING
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1 $\lim_{x \to 0} \left\{ \frac{4x^2 - \sin x}{x^3} \right\}$

Solution

By direct substitution we have $\frac{4(0)^2 - \sin(0)}{(0)^3}$ = undefined

Using L'hopitals rule

$$\lim_{x \to 0} \left\{ \frac{4x^2 - \sin x}{x^3} \right\} = \lim_{x \to 0} \left\{ \frac{8x - \cos x}{3x^2} \right\}$$

$$= \lim_{x \to 0} \left\{ \frac{8 + \sin x}{6x} \right\} = \lim_{x \to 0} \left\{ \frac{0 + \cos x}{6} \right\}$$

$$= \frac{\cos x}{6} = \frac{\cos 0}{6} = \frac{1}{6}$$

2 $y = \frac{7x^2 \cos 8x}{e^{3x}}$

$$y = \frac{uv}{w}$$

$$\frac{dy}{dx} = y \left[\frac{1}{u} \frac{du}{dx} + \frac{1}{v} \frac{dv}{dx} - \frac{1}{w} \frac{dw}{dx} \right]$$

$u = 7x^2$ $\quad v = \cos 8x$ $\quad w = e^{3x}$

$du/dx = 14x$ $\quad dv/dx = -8 \sin 8x$ $\quad dw/dx = 3e^{3x}$

$$dy/dx = y \left[\frac{1}{7x^2} \times 14x + \frac{1}{\cos 8x} \times -8 \sin 8x - \frac{1}{\cancel{e^{3x}}} \times 3\cancel{e^{3x}} \right]$$

$$\frac{dy}{dx} = y\left[\frac{2}{x} - \frac{8\sin 8x}{\cos 8x} - \frac{3}{1}\right]$$

$$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}}\left[\frac{2}{x} - \frac{8\sin 8x}{\cos 8x} - 3\right]$$

3 $y = \cos(5x^2 + 6x)$

Make $5x^2 + 6x = u$

$du/dx = 10x + 6$

$y = \cos u$

$dy/du = -\sin u$

$dy/dx = dy/du \times du/dx$

$dy/dx = -\sin u \times 10x + 6$

$dy/dx = -\sin(5x^2 + 6x)(10x + 6)$

$= -10x - 6\sin(5x^2 + 6x$

4 $\int \frac{3dx}{4x+1}$

$3\int \frac{dx}{4x+1}$

$u = 4x + 1$

$du/dx = 4$

$dx = \frac{du}{4}$

$$\int \frac{dx}{4x+1} = \frac{3du}{4u} = 3/4 \int \frac{du}{u} = \cancel{3/4 \int \frac{du \times 1}{u}}$$

$$= 3/4 \int 1/u \, du = 3/4 \ln(u)$$

$$= 3/4 \ln(4x+1) + C$$

6 $\int \frac{dx}{x^2+49} = \int \frac{dx}{x^2+7^2}$

$x = 7\tan\theta$ $\quad\quad \tan\theta = x/7 \cdot \theta = \tan^{-1}\frac{x}{7}$

$\frac{dx}{d\theta} = 7\sec^2\theta$

$dx = 7\sec^2\theta\, d\theta$

$x^2 + 7^2 = 7^2 + 7^2\tan^2\theta$

$= 49(1+\tan^2\theta)$

$1+\tan^2\theta = \sec^2\theta$

$= 49\sec^2\theta$

$x^2+7^2 = 49\sec^2\theta$

$\int \frac{\cancel{7}^{1}\cancel{\sec^2\theta}\, d\theta}{\cancel{49}_{7}\cancel{\sec^2\theta}} = \int \frac{1}{7} d\theta = 1/7 \int d\theta = 1/7[\theta] + c$

$= 1/7 \tan^{-1}\frac{x}{7} + C$

4c $\int x(x^2+9)^{1/2} dx$

$u = x^2+9$

$\frac{du}{dx} = 2x \quad dx = 1/2x\ dx$

$1/2\int \sqrt{u}\ dx$

$= 1/2\left(\frac{2u^{3/2}}{3}\right) d$

$= \frac{u^{3/2}}{3}$

$= \frac{(x^2-9)^{3/2}}{3} + C$

4C $\int e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

$= \int e^{6x}dx + \int 9x^3 dx - \int \sin 7x\,dx + \int \cos 8x\,dx$

$= 1/6 \times e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x$

$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{\cos 7x}{7} + \frac{\sin 8x}{8} + c$