

BILAMKEEN ADEDOLAPO ABDULFATIM  
MECHATRONICS ENGINEERING  
19/ENG05/019.

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1  $A = 3i + 7j - 2k$ ,  $B = i + 3j + 7k$ ,  $C = 9i + 4j + 6k$

i Angle between A and C

Solution

$$\text{Angle between A and C } \cos \theta = \frac{\bar{A} \cdot \bar{C}}{|A| |C|}$$

$$A = 3i + 7j - 2k, \quad \bar{C} = 9i - 4j + 6k$$

$$\bar{A} \cdot \bar{C} = (3i + 7j - 2k) \cdot (9i - 4j + 6k)$$

$$\bar{A} \cdot \bar{C} = 27 + 28 - 12 = -13$$

$$|A| = \sqrt{3^2 + 7^2 + 2^2} = \sqrt{62}$$

$$|C| = \sqrt{9^2 + 4^2 + 6^2} = \sqrt{133}$$

$$\cos \theta = \frac{-13}{\sqrt{62} \times \sqrt{133}} = \frac{-13}{90.8675}$$

$$\cos \theta = \frac{-13}{90.8675} \quad \theta = \cos^{-1} \left( \frac{-13}{90.8675} \right)$$

$$\theta = 98.231^\circ$$

ii Angle between B and C

$$\cos \theta = \frac{B \cdot C}{|B| |C|}$$

$$B = i + 3j + 7k, \quad C = 9i - 4j + 6k$$



$$\vec{B} \cdot \vec{C} = (i + 3j + 7k) \cdot (9i - 4j + 6k)$$

$$= 9 - 12 + 42$$

$$\vec{B} \cdot \vec{C} = 39$$

$$|\vec{B}| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$$

$$|\vec{C}| = \sqrt{9^2 + 4^2 + 6^2} = \sqrt{133}$$

$$\cos \theta = \frac{\vec{B} \cdot \vec{C}}{|\vec{B}| |\vec{C}|}$$

$$\cos \theta = \frac{39}{\sqrt{59} \times \sqrt{133}} = \frac{39}{88.5833}$$

$$\theta = \cos^{-1} \left( \frac{39}{88.5833} \right)$$

$$\theta = 63.8793^\circ$$

ii)  $u = \frac{\vec{u}}{|\vec{u}|}$

$$u = A + B + C$$

$$u = (3i + 7j + 2k) + (i + 3j + 7k) + (9i - 4j + 6k)$$

$$\vec{u} = 13i + 6j + 11k$$

$$|\vec{u}| = \sqrt{13^2 + 6^2 + 11^2} = \sqrt{169 + 36 + 121} = \sqrt{326}$$

$$u = \frac{13i + 6j + 11k}{\sqrt{326}}$$



$$2 \quad x = -8t^2; y = t^2 - 4t, z = t + 1$$

$$r = xi + yj + zk$$

$$\therefore r = 8t^2i + (t^2 - 4t)j + (t + 1)k$$

$$\frac{dr}{dt} = 16ti + (2t - 4)j + k$$

$$\frac{d^2r}{dt^2} = 16i + 2j + 0$$

$$\left| \frac{d^2r}{dt^2} \right|_{t=1} = \sqrt{16^2 + 2^2 + 0^2} =$$

$$= \sqrt{256 + 4}$$

$$= \sqrt{260}$$

$$= 2\sqrt{65}$$

$$= \cancel{16.12} = 16.1245 \text{ m/s}^2$$

$$3 \quad A = 4i + 2j - 4k, B = 8i - 2j + k, C = i + 4j - 3k$$

Find  $(A \times B) \times C$

Solution

$$A \times B = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$= i(2 - 8) - j(4 + 32) + k(-8 - 16)$$

$$= -6i - 36j - 24k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$= i(108 + 96) - j(18 + 24) + k(-24 + 36)$$

$$= 204i - 42j + 12k$$

$$(A \times B) \times C = 204i - 42j + 12k$$