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1) $\lim_{x \to 0} \left(\frac{4x^2 - \sin x}{x^3} \right)$

Using L'Hopital's Rule

$\lim_{x \to 0} \left(\frac{8x - \cos x}{3x^2} \right)$ - for the 1st derivative

$\lim_{x \to 0} \left(\frac{8 + \sin x}{6x} \right)$ - for the 2nd derivative

$\lim_{x \to 0} \left(\frac{\cos x}{6} \right) = \frac{\cos(0)}{6} = \frac{1}{6}$

2 If $y = \left(\frac{7x^2 \cos 8x}{e^{3x}} \right)$ find the derivative of y with respect to x

Solution

$y = \frac{uv}{w}$

$u = 7x^2$, $v = \cos 8x$, $w = e^{3x}$

$\frac{du}{dx} = 14x$, $\frac{dv}{dx} = -8\sin 8x$, $\frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left[\frac{1}{u} du + \frac{1}{v} dv - \frac{1}{w} dw \right]$$

$$= y\left[\frac{1}{7x^2} 14x + \frac{1}{\cos 8x}(-8\sin 8x) - \frac{1}{e^{3x}} 3e^{3x} \right]$$

$$= y\left[\frac{\cancel{14x}}{\cancel{7x^2}} + \frac{(-8\sin 8x)}{\cos 8x} - \frac{3\cancel{e^{3x}}}{\cancel{e^{3x}}} \right]$$

$$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}} \left[\frac{2}{x} - 8\tan 8x - 3 \right]$$

3. If $y = \cos(5x^2 + 6x)$, find dy/dx

Solution: Using the Chain Rule:

$y = \cos(5x^2 + 6x)$

$\frac{dy}{dx} = -\sin(5x^2 + 6x) \cdot 10x + 6$

4. Find the Integral of:

a) $\int 3dx/(4x+1)$

b) $\int dx/(x^2+49)$

c) $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

d) $\int (x\sqrt{9+x^2})\,dx$

Solution

a) $\int \frac{3}{4x+1}\,dx$

Let $u = 4x + 1$

$\frac{du}{dx} = 4$

$\frac{du}{4} = \frac{\cancel{4}dx}{\cancel{4}}$

$dx = \frac{du}{4}$

$\int \frac{3}{u}\,\frac{du}{4}$

$\frac{3}{4}\int \frac{1}{u}\,du$

Remember $\int \frac{1}{u}\,du = \ln(u)$

$\therefore \frac{3}{4}\ln(u) = \frac{3}{4}\ln(4x+1) + C$

b) $\int \frac{dx}{(x^2+49)} = \int \frac{1}{49(1+\frac{x^2}{49})} dx = \frac{1}{49} \int \frac{dx}{1+\left(\frac{x}{7}\right)^2}$

Let $u = \frac{x}{7}$

$\frac{du}{dx} = \frac{1}{7}$

$dx = 7\,du$

$\frac{1}{49} \int \frac{7\,du}{1+u^2} = \frac{7}{49} \int \frac{du}{1+u^2} = \frac{1}{7} \int \frac{du}{1+u^2}$

Recall: $\int \frac{1}{1+u}\,du = \arctan u$

$= \frac{1}{7} \int \frac{1}{1+u^2}\,du = \frac{\arctan\left(\frac{x}{7}\right)}{7} + C$

c) $\int e^{6x} + 9x^3 - \sin 7x + \cos 8x\,dx$

$\left[\frac{e^{6x}}{6} + \frac{9x^{3+1}}{3+1} - \left(\frac{-\cos 7x}{7}\right) + \frac{\sin 8x}{8}\right] + C$

$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{\cos 7x}{7} + \frac{\sin 8x}{8} + C$

$\int x\sqrt{9+x^2}\,dx$

Let $u = 9+x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \cancel{x}\sqrt{u}\,\frac{du}{2\cancel{x}} = \int u^{1/2}\,\frac{du}{2} = \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\,\frac{u^{3/2}}{3/2} = \frac{u^{3/2}}{2 \cdot 3/2} = \frac{u^{3/2}}{3} = \frac{(9+x^2)^{3/2}}{3}$ //