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19/sci01/034

computer science

1) If $A = 3i + 7j - 2k$, $B = i + 3j + 7k$, $C = 9i - 4j + 6k$, find the angle between

i) A and C

$$\vec{A} \cdot \vec{C} = (3i + 7j - 2k) \cdot (9i - 4j + 6k)$$

$$= 27 - 28 - 12$$

$$= -13$$

$$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{62}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{C}}{|A||C|}$$

$$\cos \theta = \frac{-13}{\sqrt{62} \times \sqrt{133}}$$

$$\cos \theta = -0.1432$$

$$\theta = \cos^{-1}(-0.1432)$$

$$= 98.23^\circ$$

ii) B and C

$$B \cdot C = (i + 3j + 7k) \cdot (9i - 4j + 6k)$$

$$= 9 - 12 + 42$$

$$= 39$$

$$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$\cos \theta = \frac{B \cdot C}{|B||C|}$$

$$\cos \theta = \frac{39}{\sqrt{59} \times \sqrt{133}}$$

$$\cos \theta = 0.4403$$

$$\theta = \cos^{-1}(0.4403)$$

$$= 63.9^\circ$$

11) the unit vector in the direction of $(A+B+C)$

$$\hat{c}_{A+B+C} = \frac{A+B+C}{|A+B+C|}$$

$$A+B+C = (3i+7j-2k) + (i+3j+7k) + (9i-4j+6k) \\ = 13i+6j+11k$$

$$|A+B+C| = \sqrt{13^2+6^2+11^2} = \sqrt{326}$$

$$\hat{c}_{A+B+C} = \frac{13i+6j+11k}{\sqrt{326}} \\ = \frac{13}{\sqrt{326}}i + \frac{6}{\sqrt{326}}j + \frac{11}{\sqrt{326}}k$$

2 $x = -8t^2$

$$y = t^2 - 4t$$

$$z = t+1$$

$$r = (-8t^2)i + (t^2-4t)j + (t+1)k$$

$$\frac{dr}{dt} = (-16t)i + (2t-4)j + k$$

$$\text{acceleration } \frac{d^2r}{dt^2} = -16i + 2j$$

$$\left| \frac{d^2r}{dt^2} \right| = \sqrt{(-16)^2 + 2^2} = 16.12$$

3 $A = 4i + 2j - 4k$

$$B = 8i - 2j + k$$

$$C = i + 4j - 3k$$

$$(A \times B) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$= i \begin{vmatrix} 2 & -4 \\ -2 & 1 \end{vmatrix} - j \begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k \begin{vmatrix} 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$= i(2-8) - j(4+32) + k(-8-16)$$

$$= -6i - 36j - 24k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$= i \begin{vmatrix} -36 & -24 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} -6 & -24 \\ 1 & -3 \end{vmatrix} + k \begin{vmatrix} -6 & -36 \\ 1 & 4 \end{vmatrix}$$

$$= i(108 + 96) - j(18 + 24) + k(-24 + 36)$$

$$= 204i - 42j + 12k$$