MAT 104

Using L'hopital's rule

$$\lim_{x \to 0} \left\{ \frac{4x^2 - \sin x}{x^3} \right\} = \lim_{x \to 0} \left\{ \frac{8x - \cos x}{3x^2} \right\}$$

$$= \lim_{x \to 0} \left\{ \frac{8 + \sin x}{6x} \right\} = \lim_{x \to 0} \left\{ \frac{\cos x}{6} \right\}$$

Substituting $x = 0$

$$\lim_{x \to 0} \left\{ \frac{4x^2 - \sin x}{x^3} \right\} = \frac{\cos(0)}{6} = \frac{1}{6}$$

$$y = \frac{7x^2 \cos 8x}{e^{3x}}$$

let $u = 7x^2 \cos 8x$

$v = e^{3x}$ $\quad \frac{dv}{dx} = 3e^{3x}$

$$\frac{du}{dx} = a\frac{db}{dx} + B\frac{da}{dx}$$

where $a = 7x^2$ $\quad \frac{da}{dx} = 14x$ $\quad B = \cos 8x$ $\quad \frac{db}{dx} = -8\sin 8x$

$$\frac{du}{dx} = 7x^2(-8\sin 8x) + \cos 8x\,(14x)$$

$$\frac{du}{dx} = -56x^2 \sin 8x + 14x \cos 8x$$

$$\frac{dy}{dx} = \frac{v\,du/dx - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{e^{3x}(-56x^2 \sin 8x + 14x \cos 8x) - 7x^2 \cos 8x (3e^{3x})}{(e^{3x})^2}$$

$$\frac{dy}{dx} = \frac{e^{3x}(-56x^2 \sin 8x + 14x \cos 8x - 21x^2 \cos 8x)}{(e^{3x})^2}$$

$$\frac{dy}{dx} = \frac{-56x^2 \sin 8x + 14x \cos 8x - 21x^2 \cos 8x}{e^{3x}}$$

$$\frac{dy}{dx} = e^{-3x}(-56x^2 \sin 8x + 14x \cos 8x - 21x^2 \cos 8x)$$

3. $y = \cos(5x^2 + 6x)$

Remember $\cos ax = a \cos ax$

$$\frac{dy}{dx} = 10x + 6\,(-\sin(5x^2 + 6))$$

$$\frac{dy}{dx} = -(10x + 6 \sin(5x^2 + 6))$$

4 (a) $\int \frac{3dx}{4x+1} = 3\int \frac{dx}{4x+1}$

let $u = 4x+1$ $\quad du/dx = 4$

$du/4 = dx$

$$3\int \frac{1}{4x+1} \cdot du/4 = 3/4 \int \frac{1}{4x+1}$$

$$= \frac{3}{4} \ln(4x+1)$$

[illegible]

$x = 7\tan\theta$ [illegible]

[illegible]

$49 + x^2 = 49 + (7\tan\theta)^2$

$49 + x^2 = 49 + 49\tan^2\theta$

$49 + x^2 = 49(1 + \tan^2\theta)$

Recall $\sec^2\theta = 1 + \tan^2\theta$

$49 + x^2 = 49\sec^2\theta$

$\int \frac{dx}{x^2+49} = \int \frac{7\sec^2\theta}{49\sec^2\theta} = \frac{1}{7}\int d\theta$

$\frac{1}{7}[\theta] = \frac{1}{7}\tan^{-1}\left(\frac{x}{7}\right) + C$

(2) $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx = \frac{1}{6}e^{6x} + \frac{9}{4}x^4 + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x$

$= \frac{1}{6}e^{6x} + \frac{9}{4}x^4 + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$

(3) $\int x\cdot(9+x^2)^{1/2}\,dx$ let $u = 9+x^2$ $\quad du/dx = 2x$

$2x\,dx = du$

$\int x\cdot(9+x^2)^{1/2} = \int x \cdot u^{1/2} \cdot du/2x$

$= \frac{1}{2}\int du\,\sqrt{u} = \frac{1}{2}\left[u^{1/2+1}/(1/2+1)\right] + C$

$= \frac{1}{2}\left[u^{3/2}/(3/2)\right] + C = \frac{1}{2}\left[u^{3/2} \times \frac{2}{3}\right]$

$u^{3/2}/3 + C = \frac{1}{3}u^{3/2} = \frac{1}{3}(9+x^2)^{3/2}$