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MAT102

i. $A = 3i + 7j - 2k$, $B = i + 3j + 7k$, $C = 9i - 4j + 6k$

ii. $A \cdot C = |A||C| \cos \theta$

$$\cos \theta = \frac{A \cdot C}{|A||C|}$$

$$A \cdot C = 27 - 28 - 12 = -13$$

$$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{62}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$\cos \theta = \frac{-13}{\sqrt{62} \sqrt{133}}$$

$$\cos \theta = -0.1432$$

$$\theta = 98.23^\circ$$

iii. B and C

$$B \cdot C = 9 - 12 + 42 = 39$$

$$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$$

$$|C| = \sqrt{133}$$

$$\cos \theta = \frac{39}{\sqrt{59} \times \sqrt{133}}$$

$$\cos \theta = 0.4403$$

$$\theta = 63.88^\circ$$

iii) The unit vector in the direction $(A+B+C)$

$$(3i + 7j - 2k) + (i + 3j + 7k) + (9i - 4j + 6k)$$

$$= 13i + 6j + 11k$$

$$e_{A+B+C} = \frac{A+B+C}{|A+B+C|}$$

$$|A+B+C| = \sqrt{13^2 + 6^2 + 11^2} = \sqrt{326}$$

$$e_{A+B+C} = \frac{13i + 6j + 11k}{\sqrt{326}}$$

$$e_{A+B+C} = \frac{13i}{\sqrt{326}} + \frac{6j}{\sqrt{326}} + \frac{11k}{\sqrt{326}}$$

2. A particle moves along a curve $x = 8t^2 - 4t$, $z = t + 1$ where t is time find the modulus of acceleration at $t = 1$

Soln

$$\vec{r} = 8t^2 \mathbf{i} + (t^2 - 4t) \mathbf{j} + (t + 1) \mathbf{k}$$

$$\frac{d\vec{r}}{dt} = 16t \mathbf{i} + (2t - 4) \mathbf{j} + \mathbf{k} = \text{velocity}$$

$$\left| \frac{d^2 \vec{r}}{dt^2} \right| = 16 \mathbf{i} + 2 \mathbf{j} = \text{acceleration} = \sqrt{16^2 + 2^2}$$

$$= \sqrt{256 + 4} = \sqrt{260} = 16.12 \text{ m/s}^2$$

3. $A = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$, $B = 8\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $C = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
 $(A \times B) \times C$

$$(A \times B) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$\begin{aligned} (A \times B) &= [2 - (-4 \times -2)] \mathbf{i} - [4 - (8 \times -4)] \mathbf{j} + [-8 - (2 \times 8)] \mathbf{k} \\ &= (2 - 8) \mathbf{i} - (4 + 32) \mathbf{j} + (-8 - 16) \mathbf{k} \\ &= -6 \mathbf{i} - 36 \mathbf{j} - 24 \mathbf{k} \end{aligned}$$

$$(A \times B) \times C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$\begin{aligned} (A \times B) \times C &= [(-36 \times -3) - (-24 \times 4)] \mathbf{i} - [(-6 \times -3) - (-24 \times 1)] \mathbf{j} + [(-6 \times 4) - (-36 \times 1)] \mathbf{k} \\ &= [108 + 96] \mathbf{i} - [18 + 24] \mathbf{j} + [-24 + 36] \mathbf{k} \\ &= 204 \mathbf{i} - 42 \mathbf{j} + 12 \mathbf{k} \end{aligned}$$