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Reg No: 19/EALG05/042

Course Code: Mat 104

Course Title: ~~General~~ Mathematics III

1) Find the limit of the function $\left[\frac{e^{4x^2} - \sin x}{x^3} \right]$
as $x \rightarrow 0$

Solution

$$\lim_{x \rightarrow 0} \frac{4x^2 - \sin x}{x^3}$$

$$\frac{8x - \cos x}{3x^2}$$

$$\frac{8 + \sin x}{6}$$

$$\lim_{x \rightarrow 0} \frac{8 + \sin x}{6} = \frac{1}{6}$$

$$\therefore \lim_{x \rightarrow 0} \frac{4x^2 - \sin x}{x^3} = \frac{1}{6}$$

2) If $y = \left(\frac{7x^2 \cos 8x}{e^{3x}} \right)$ Find the derivative of y

with respect to x

Solution

$$\frac{dy}{dx} = y \left(\frac{1}{v} \frac{dv}{dx} + \frac{1}{w} \frac{dw}{dx} \right)$$

$$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}} \left(\frac{1}{7x^2} (14x) + \frac{1}{\cos 8x} (-8 \sin 8x) \right)$$

$$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}} \left(\frac{1}{7x^2} (14x) + \frac{1}{\cos 8x} (-8 \sin 8x) - \frac{1}{e^{3x}} (3e^{3x}) \right)$$

$$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}} \left(\frac{2}{x} - 8 \tan 8x - 3 \right)$$

3) If $y = \cos(5x^2 + 6x)$, find dy/dx

Solution

$$y = \cos(5x^2 + 6x)$$

$$v = 5x^2 + 6x \quad y = \cos v$$

$$\frac{dv}{dx} = 10x + 6 \quad \frac{dy}{dx} = -\sin v$$

$$\frac{dy}{dx} = -(10x + 6) \sin(5x^2 + 6x)$$

4) Find the Integral of the Following

a) $\int \frac{3}{4x+1} dx$

b) $\int \frac{1}{4x^2+49} dx$

c) .

d) $\int x \sqrt{9+x^2} dx$

Solution

a) $\int \frac{3 dx}{4x+1}$

$$v = 4x+1$$

$$\frac{dv}{dx} = 4$$

$$dx = \frac{dv}{4}$$

$$dx = \frac{dv}{4}$$

$$4$$

let $u = 3 dx$

let $v = 4x+1$

$$\int \frac{3}{v} \frac{dv}{4}$$

$$\frac{3}{4} (v) = \frac{3}{4} \ln(4x+1) + C$$

b) $\int \frac{dx}{x^2+49}$

$$x = 7 \tan \theta \quad x^2 = 49 \tan^2 \theta$$

$$\frac{dx}{d\theta} = 7 \sec^2 \theta \quad d\theta$$

$$dx = 7 \sec^2 \theta$$

$$x^2 + 49 = 49 \tan^2 \theta + 49 = 49 (1 + \tan^2 \theta)$$

$$x^2 + 49 = 49 \sec^2 \theta$$

$$= \int \frac{1}{7} [\theta] d\theta$$

$$\frac{1}{7} \tan^{-1} \frac{x}{7} + C$$

c) $\int e^{6x} (9x^3 - \sin 7x + \cos 8x) dx$

$$\frac{1}{6} e^{6x} \frac{9x^3 \cdot 4}{9} + \frac{1}{7} \cos 7x + \frac{1}{8} \sin 8x + C$$

d) $\int x \sqrt{9+x^2} dx$

$$v = 9+x^2$$

$$\frac{dv}{dx} = 2x$$

$$dx$$

$$\int v^{1/2} \frac{dv}{2x}$$

$$\int \frac{v^{1/2}}{2} dv$$

$$\int \frac{3/2}{2 \cdot \frac{3}{2}} = \frac{v^{3/2}}{3}$$

$$\int x \sqrt{4+x^2} dx = \frac{(4+x^2)^{3/2}}{3} + C$$