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1. The following non-linear speed-density relationship is assumed for a highway link:

$$u = 0.001(k - 250)^2 - 1.4$$

   Where u has units of km/h and k has units of vehicles per km.

   Estimate the free-flow speed, the jam density, the speed and density at maximum flow and the lane capacity of the link in question.

Solution

Free-flow speed is speed at zero density; therefore, putting k=0

$u = 0.001(250)^2 - 1.4 = 61.1 km\ h^{-1}$

Jam density is density at zero speed, therefore putting u= 0

0= $0.001(k - 250)^2 - 1.4 = 61.1$

0= $0.001(k^2 - 500k + 62500)^2 - 1.4$

$0.001k^2 - 0.5k + 62.5 - 1.4 = 0$

$0.001k^2 - 0.5k + 61.1 = 0$

Solving the quadratic equation:

K =287 or 212 veh/km (287 veh/km chosen)

$q = uk,\ therefore$

$q = (0.001k^2 - 0.5k + 61.1)\ k$

$q = 0.001k^3 - 0.5k^2 + 61.1k$

At max q, $\frac{dq}{dk} = 0,\ therefore,$

$0.003k^2 - k + 61.1 = 0$

Solving this quadratic equation

$k_{qmax} = 252.75 \ or \ 80.58 \ veh/km$ (chosen 80.58 veh/km)

If k = 80.58 veh/km we can solve for u as follows

$u_{qmax} = 0.001(80.58 - 250)^2 - 1.4 = 27.30 \ km \ h^{-1}$

Therefore,

$q_{max} = 80.58 \times 27.30 = 2199.83 \ veh \ h^{-1}$

2. The following flow-density relationship is assumed for a highway link: q + 60u(ln u) =250u

   Estimate the free-flow speed, the speed at maximum flow, the maximum flow on the link

   In question and density and maximum flow

3. At a given highway location, assuming that vehicle arrivals are Poisson distributed, vehicles are counted in intervals of 30 seconds.

   100 such counts are taken and it is noted that no vehicle arrives in 15 of these 100 intervals

   (a) In how many of these 100 intervals will 3 cars arrive?

   (b) Estimate the percentage of time that headways will be 6 seconds or greater

   (c) Estimate the percentage of time that headways will be less than 4 seconds

Solution

a) $t = 30s$

$$p(0) = \frac{15}{100} = 0.15$$

$$p(n) = \frac{(qt)^n \ e^{-qt}}{n!}$$

$$p(0) = \frac{(qt)^0 e^{-qt}}{0!} = 0.15$$

Therefore

$$e^{-qt} = 0.15$$

$$-qt = In(0.15) = -1.89712$$

$$q = 1.89712 + 30 = 0.0632 veh/s$$

Therefore

$$p(3) = \frac{(0.0632 \times 30)^3 e^{-(0.0632 \times 30)}}{3!}$$

$$= \frac{(1.897)^3 e^{-(1.897)} = 0.171}{6}$$

There is thus a 17.1% chance that 3 vehicles will arrive within any one interval. Therefore, over the 100 counts taken, assuming that traffic is Poisson distributed, 3 cars will arrive in just greater than 17 of these:

b) $P(headway \geq 6) = e^{-(0.0632 \times 6)}$

$= 0.684\ 0r\ 68.4\%\ of\ the\ time$

c) $P(headway \geq 4) = e^{-(0.0632 \times 6)} = 0.777\ or\ 77.7\%\ of\ time$

Therefore,

$$\begin{aligned} P(headway \geq 4) &= 1 - P(headway \geq 4) \\ &= 1 - 0.777 \\ &= 0.223\ or\ 22.3\%\ of\ the\ time \end{aligned}$$