

Analysis of Mathematical Goodness

Computer Engineering

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Maths 102 assignment

1) A particle moves along a curve $x = 8t^3$, $y = 4t^3 - 7t$ and $z = t + 3$, where t is time. Find its

i) velocity

ii) acceleration

Solution:

$$r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$r = 8t^3\mathbf{i}$$

$$r = (8t^3)\mathbf{i} + (4t^3 - 7t)\mathbf{j} + (t + 3)\mathbf{k}$$

$$\text{i) velocity} = \frac{dr}{dt} = 24t^2\mathbf{i} + (12t^2 - 7)\mathbf{j} + \mathbf{k}$$

$$\text{ii) acceleration} = \frac{d^2r}{dt^2} = 48t\mathbf{i} + 24\mathbf{j}$$

2) Find the unit tangent vector to the space curve

$$x = 3t, y = 3t^3 \text{ and } z = t^2 \text{ at } t = 1$$

Solution:

$$r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$r = 3t\mathbf{i} + 3t^3\mathbf{j} + t^2\mathbf{k}$$

$$\frac{dr}{dt} = 3\mathbf{i} + 9t^2\mathbf{j} + 2t\mathbf{k}$$

$$\text{at } t = 1$$

$$\frac{dr}{dt} = 3\mathbf{i} + 9\mathbf{j} + 2\mathbf{k}$$

$$|\frac{dr}{dt}| = \sqrt{3^2 + 3^2 + 2^2} = \sqrt{9+9+4}$$

$$|\frac{dr}{dt}| = \sqrt{22}$$

$$\mathbf{T} = \frac{\frac{dr}{dt}}{|\frac{dr}{dt}|} = \frac{3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}}{\sqrt{22}}$$