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Assignment

1) $\lim_{x \to 0} \left(\frac{4x^2 - \sin x}{x^3} \right)$

Using L'Hopital's Rule

$\lim_{x \to 0} \left(\frac{8x - \cos x}{3x^2} \right)$ - for 1st derivative

$\lim_{x \to 0} \left(\frac{8 + \sin x}{6x} \right)$ - 2nd derivative

$\lim_{x \to 0} \left(\frac{\cos x}{6} \right) = \frac{\cos(0)}{6} = \frac{1}{6}$

2) $y = \left(\frac{7x^2 \cos 8x}{e^{3x}} \right)$

Soln

$y = \frac{uv}{w}$ $u = 7x^2$, $v = \cos 8x$, $w = e^{3x}$

$\frac{du}{dx} = 14x$, $\frac{dv}{dx} = -8\sin 8x$, $\frac{dw}{dx} = 3e^{3x}$

$\frac{dy}{dx} = y\left[\frac{1}{u} du + \frac{1}{v} dv - \frac{1}{w} dw \right]$

$= y\left[\frac{1}{7x^2} 14x + \frac{1}{\cos 8x}(-8\sin 8x) - \frac{1}{e^{3x}} 3e^{3x} \right]$

$= y\left[\frac{14x^2}{7x^2} + \frac{(-8\sin 8x)}{\cos 8x} - \frac{3e^{3x}}{e^{3x}} \right]$

$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}} \left[\frac{2}{x} - 8\tan 8x - 3 \right]$

c.) $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

$$\left[\frac{e^{6x}}{6} + \frac{9x^{3+1}}{3+1} - \frac{(-\cos 7x)}{7} + \frac{\sin 8x}{8}\right] + c$$

$$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{\cos 7x}{7} + \frac{\sin 8x}{8} + c$$

d.) $\int x\sqrt{9+x^2}\,dx$

$u = 9 + x^2$, $du/dx = 2x$

$dx = du/2x$

$\int x\sqrt{9+x^2}\,dx = \int x\sqrt{u}\,du/2x$

$= \int u^{1/2}\frac{du}{2}$

$= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2}$

$= \frac{u^{3/2}}{2 \cdot 3/2} = \frac{u^{3/2}}{3}$

$= \frac{(9+x^2)^{3/2}}{3}$

3) $y = \cos(5x^2 + 6x)$, find dy/dx

Soln

Using the Chain Rule

$y = (\cos 5x^2 + 6x)$

$\frac{dy}{dx} = -\sin(5x^2 + 6x) \cdot 10x + 6$

4a) $\int \frac{3}{4x+1}\, dx$

Let $u = 4x + 1$

$\frac{du}{dx} = 4$

$dx = du/4$

$\int \frac{3}{u} \frac{du}{4}$

$\frac{3}{4} \int \frac{1}{u} du$

Recall, $\int \frac{1}{u} du = \ln(u)$

$\therefore \frac{3}{4} \ln(u) = \frac{3}{4} \ln(4x+1)$

b) $\int \frac{dx}{(x^2+49)} = \int \frac{1}{4(1+\frac{x^2}{49})} dx = \frac{1}{49} \int \frac{dx}{1+(\frac{x}{7})^2}$

let $u = x/7$, $dy/dx = 1/7$

$dx = 7\,du$

$\frac{1}{49} \int \frac{7\,du}{1+u^2} = \frac{7}{49} \int \frac{du}{1+u^2} = \frac{1}{7} \int \frac{du}{1+u^2}$

Recall, $\int \frac{1}{1+u} du = \arctan u$

$= \frac{1}{7} \int \frac{1}{1+u^2} du = \frac{\arctan(x/7)}{7}$