DEPT: COMPUTER ENGINEERING

1, $x^{1/2} \ln x$

2, $3\cos 6t \cos t$

3, $\sin^3 x \cos^4 x$

Solution

(1, $x^{1/2} \ln x$

$\int x^{1/2} \ln x$

$u = x^{1/2}$ $dv = \ln x$

$\frac{dy}{dx} = 1/2 x^{-1/2}$

$\int V \frac{du}{dx} + \int u \frac{dv}{dx}$

$\ln x \int \frac{x^{1/2}}{dx} + x^{1/2} \int \frac{d \ln x}{dx}$

$\ln x \left[\frac{x^{3/2}}{3/2} \right] + x^{1/2} \left[\int \frac{1}{x} \right] + C$

$\frac{2 \ln x}{3} \cdot x^{3/2} + \frac{x^{1/2}}{x} + C$

$\frac{2 x^{3/2} \ln x}{3} + \frac{\sqrt{x}}{x} + C$

2) $\int \sin^3 x \cos^4 x$

(3) $\int \sin^3 x \cos^4 x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$

$\int \cos^4 x \sin^3 x \, dx$

$\int u^4 \sin^3 x \cdot \frac{-du}{\sin x}$

$-\int u^4 \sin x \cdot du$

$-\int u^4 \sin^2 x \cdot du$

Recall that

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$-\int \sin^2 x \, u^4 \, du$

$-\int (1 - \cos^2 x) u^4 \, du$

but $u = \cos x$

$-\int (1 - u^2) u^4 \, du$

$-\int (u^4 - u^6) \, du$

$-\left[\frac{u^{4+1}}{4+1} - \frac{u^{6+1}}{6+1}\right] + c$

$-\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + c$

$\frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c$

(2) $\int 2\int \cos 6t \cos t \, dt = 12 \int \cos 6t \cos t \, dt$

~~let~~ $A = 6t \quad B = 1t$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= \frac{1}{2}[\cos(6+1) + \cos(6-1)]$

$= \frac{1}{2}[\cos 7t + \cos 5t]$

$$\int 2\cos 6t \cos t\, dt = \tfrac{1}{2}\int (2\cos 7t + \cos 5t)$$
$$= \frac{1}{2}\left[\frac{2\sin 7t}{7} - \frac{\sin 5t}{5}\right]$$
$$= \frac{\sin 7t}{7} - \frac{\sin 5t}{5} + c$$

3) $\int \sin^3 x \cos^4 x = \int \sin^m x \cos^n x$

Since m is odd

$u = (\cos^2 x)^2$

$du/dx = -\sin x \Rightarrow dx = -du/\sin$

3) $\int \sin^3 x \cos^4 x\, dx$

$\int \sin^3 x ((\cos^2 x)^2\, dx$

$u = (\cos^2 x)^2$

du/dx