1 $\lim_{x \to 0} \dfrac{4x^2 - \sin x}{x^3}$

By direct substitution we have

$\dfrac{4(0)^2 - \sin(0)}{(0)^3} = \dfrac{0 - 0}{0} = 0$

2. $y = \dfrac{7x^2 \cos 8x}{e^{3x}}$

$\dfrac{dy}{dx} = y\left[\dfrac{1}{u}\dfrac{du}{dx} + \dfrac{1}{v}\dfrac{dv}{dx} - \dfrac{1}{w}\dfrac{dw}{dx}\right]$

$u = 7x^2$ $\quad v = \cos 8x$ $\quad w = e^{3x}$

$\dfrac{du}{dx} = 14x$ $\quad \dfrac{dv}{dx} = -8\sin 8x$ $\quad \dfrac{dw}{dx} = 3e^{3x}$

$\dfrac{dy}{dx} = y\left[\dfrac{1}{7x^2}(14x) + \dfrac{1}{\cos 8x}(-8\sin 8x) - \dfrac{1}{e^{3x}}(3e^{3x})\right]$

$\dfrac{dy}{dx} = y\left[\dfrac{2}{x} - 8\tan 8x - 3\right]$

$\dfrac{dy}{dx} = \dfrac{7x^2\cos 8x}{e^{3x}}\left[\dfrac{2}{x} - 8\tan 8x - 3\right]$

3. $y = \cos(5x^2 + 6x)$

let $u = 5x^2 + 6x$ $\quad y = \cos u$

$\dfrac{du}{dx} = 10x + 6$ $\quad \dfrac{dy}{du} = -\sin u$

$\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$

$\dfrac{dy}{dx} = (10x + 6)\left(-\sin(5x^2 + 6x)\right]$

4a $\frac{3}{4x+1}\,dx$

let $u = 4x+1$

$\frac{du}{dx} = 4$, $\frac{dx}{du} = \frac{1}{4}$

$\int \frac{3}{u} \frac{dx}{du} \cdot du$

$\frac{1}{4}\int \frac{3}{4x+1}$

Recall $\int 1/x = \ln x + c$

$= \frac{3}{4}\ln(4x+1) + c$

b $\int \frac{dx}{x^2+49}$

Recall $\int \frac{dx}{a^2+x^2}$

$= \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$a = 7$

$\therefore \int \frac{dx}{x^2+49} = \frac{1}{7}\tan^{-1}\frac{x}{7} + c$

c $(e^{6x} + 9x^3 - \sin 7x + \cos 8x)dx$

$\int \cancel{(e^6} (e^{6x} + 9x^3 - \sin 7x + \cos 8x)dx$

$= \frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x$

d) $\int x\sqrt{9+x^2}\,dx$

let $u = 9+x^2$

$\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$

$\int x(u)^{1/2}\, du/2x$

Recall $u = 9+x^2$

$x^2 = u-9$

$x = \sqrt{u-9}$

$\int x(u)^{1/2}\, du/2x$

$\int (u)^{1/2}\, du/2$

$\frac{1}{2}\int (u)^{1/2}\,du$

$\frac{1}{2}\left[\frac{u^{1/2+1}}{1/2+1}\right] + c$

$\frac{1}{2}\cdot\frac{2}{3}\left[9+x^2\right]^{3/2} + c$

$\frac{1}{3}(9+x^2)^{3/2} + c$.