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1. The following non-linear speed-density relationship is assumed for a highway link:

$$u=0.001(k-250)^{2}-1.4 $$

Where u has units of km/h and k has units of vehicles per km.

Estimate the free-flow speed, the jam density, the speed and density at maximum flow and the lane capacity of the link in question.

Solution

Free-flow speed is speed at zero density; therefore, putting k=0

$$u=0.001(250)^{2}-1.4=61.1km h^{-1}$$

Jam density is density at zero speed, therefore putting u= 0

0$=0.001(k-250)^{2}-1.4=61.1$

0$=0.001(k^{2}-500k+62500)^{2}-1.4$

$$0.001k^{2}-0.5k+62.5-1.4=0$$

$$0.001k^{2}-0.5k+61.1=0$$

Solving the quadratic equation:

K =287 or 212 veh/km (287 veh/km chosen)

$$q=uk, therefore$$

$$q=(0.001k^{2}-0.5k+61.1) k$$

$$q=0.001k^{3}-0.5k^{2}+61.1k$$

At max q, $\frac{dq}{dk}=0, therefore,$

$$0.003k^{2}-k+61.1=0$$

Solving this quadratic equation

$k\_{qmax}=252.75 or 80.58 veh/km$ (chosen 80.58 veh/km)

If k = 80.58 veh/km we can solve for u as follows

$$u\_{qmax}=0.001(80.58-250)^{2}-1.4=27.30 km h^{-1}$$

Therefore,

$$q\_{max}=80.58 × 27.30=2199.83 veh h^{-1}$$

1. The following flow-density relationship is assumed for a highway link: q + 60u(ln u) =250u

       Estimate the free-flow speed, the speed at maximum flow, the maximum flow on the link

 In question and density and maximum flow

Solution

q + 60u(ln u) =250u

$$jam density is density at zero, therefore u=0$$

$$q+60\left(lnu^{2}\right)=250u$$

$$q+60u^{2}=e^{250u}$$

$$q+60\left(0\right)^{2}=e^{250\left(0\right)}$$

$$q=1$$

At max $q=\frac{dq}{du}=0$

$$q+60Inu^{2}=250u$$

$$q=250u-60Inu²$$

$$\frac{dq}{du}=250-60\left(\frac{2}{u}\right)$$

$$250-\frac{120}{u}=0$$

$$U\_{max}=\frac{120}{250}$$

$$U\_{max}=0.48 km h^{-1}$$

$$q=uk$$

$$1=0.48km h^{-1}$$

$$k=\frac{1}{0.48}=2veh/km$$

1. At a given highway location, assuming that vehicle arrivals are Poisson distributed, vehicles are counted in intervals of 30 seconds.

100 such counts are taken and it is noted that no vehicle arrives in 15 of these 100 intervals

(a) In how many of these 100 intervals will 3 cars arrive?

(b) Estimate the percentage of time that headways will be 6 seconds or greater

(c) Estimate the percentage of time that headways will be less than 4 seconds

Solution

1. $t=30s$

$$p\left(0\right)=\frac{15}{100}=0.15$$

$$p\left(n\right)=\frac{(qt)^{n} e^{-qt}}{n!}$$

$$p\left(0\right)=\frac{(qt)^{0}e^{-qt}}{0!}=0.15$$

Therefore

$$e^{-qt}=0.15$$

$$-qt=In\left(0.15\right)=-1.89712$$

$$q=1.89712+30=0.0632veh/s$$

Therefore

$$p\left(3\right)=\frac{\left(0.0632×30\right)^{3}e^{-\left(0.0632×30\right)}}{3!}$$

$$=\frac{\left(1.897\right)^{3}e^{-\left(1.897\right)}=0.171}{6}$$

There is thus a 17.1% chance that 3 vehicles will arrive within any one interval. Therefore, over the 100 counts taken, assuming that traffic is Poisson distributed, 3 cars will arrive in just greater than 17 of these:

1. $P\left(headway\geq 6\right)=e^{-\left(0.0632×6\right)}$

$$=0.684 0r 68.4\% of the time$$

1. $P\left(headway\geq 4\right)=e^{-\left(0.0632×6\right)}=0.777 or 77.7\% of time$

Therefore,

$$P\left(headway\geq 4\right)=1-P\left(headway\geq 4\right)$$

 $=1-0.777$

$ $ $=0.223 or 22.3\% of the time$