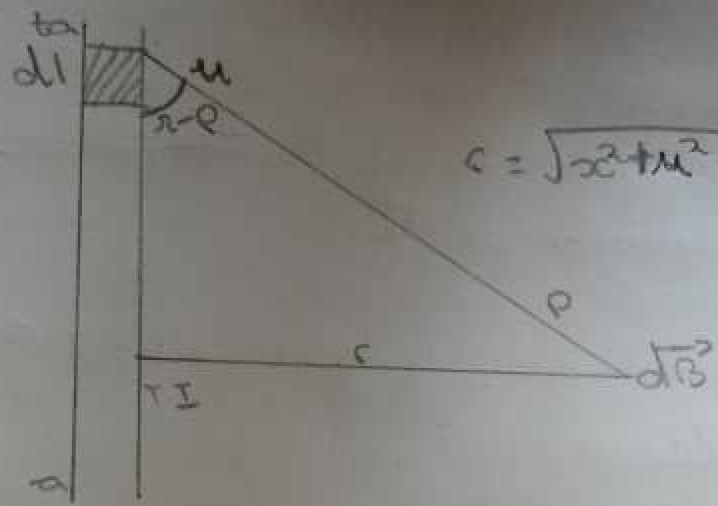
5b $B = \frac{\mu_0 I}{2\pi r}$

Soln



$r = \sqrt{x^2 + u^2}$ pythagoras theorem

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - u)}{x^2 + y^2} \quad — *$$

But $\sin(\pi - u) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{\frac{1}{2}}} \quad — **$

Sub ** into *

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl\, x}{(x^2+y^2)(x^2+y^2)^{\frac{1}{2}}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl\, x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

Section B

(4a) What is Magnetic flux?

This is the dot product of the area and the magnetic field passing through it. It is defined as the strength of the magnetic field which can be represented by line of forces. It is representeby by the symbol Φ given as $\Phi = B \cdot dA$.

(b) Soln

given $m = 9.11 \times 10^{-31}$ kg, $r = 1.4 \times 10^{-7}$ m, $B = 3.5 \times 10^{-1}$ weber/meter^2

$q = 1.6 \times 10^{-19}$ C

$$w = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = \frac{5.6 \times 10^{-20}}{9.11 \times 10^{-31}}$$

$$w = 6.15 \times 10^{10} T_z^{-1}$$

(c) Discuss the answer in 4b

We are asked to find the Cyclotron frequency of the following electron which is also called "angular speed" because the charge particle circulates at this angular frequency or angular speed called "Cyclotron".

Recall $w = \frac{v}{r} = \frac{qB}{m}$

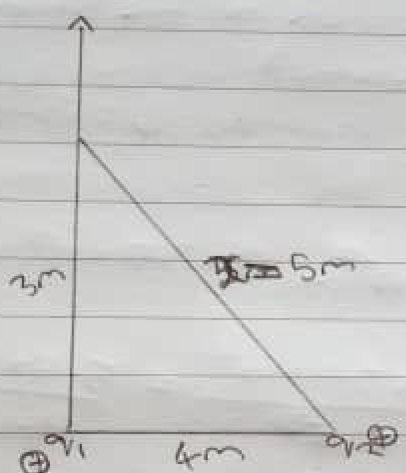
Substituting the variables we have;

$$w = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = 6.15 \times 10^{10} T_z^{-1}$$

$\therefore$ The cyclotron frequency is $6.15 \times 10^{10} T^{-1}$.

i) The electric field at a point Q on the y axis at y=3m due to the charges.

Soln



$$x = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5m$$

$$E_1 = \frac{kq_1}{r_1^2} = \frac{9\times10^9 \times 8\times10^{-9}}{(3)^2} = 8 \text{ N/C}$$

$$E_2 = \frac{kq_2}{r_2^2} = \frac{9\times10^9 \times 12\times10^{-9}}{(5)^2} = 4.32 \text{ N/C}$$

$$\vec{E}_{net} = \vec{E_1} + \vec{E_2} = 8 + 4.32 = 12.32 \text{ N/C}$$

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vector	Angle	x-component	y-component
$E_{q_1} = 59504\ N/C$	$63°$	$-59504\cos 63°$ $= -27014.3\ N/C$	$59504\sin 63°$ $= 53018.5\ N/C$
$E_{q_2} = 59504\ N/C$	$63°$	$-59504\cos 63°$ $= -27014.3\ N/C$	$59504\sin 63°$ $= 53018.5\ N/C$
$E_Q = 7.4\times10^9 Q\ N/C$	$90°$	$7.4\times10^9 Q\cos 90°$ $= 0\ N/C$	$7.4\times10^9 Q\sin 90°$ $= 7.4\times10^9 Q\ N/C$
		$E_{fx} = 0\ N/C$	$E_{fy} = 106037 + 7.4\times10^9 Q\ N/C$

$$\text{magnitude} = \sqrt{(E_{fx})^2 + (E_{fy})^2}$$

$$E_q = \sqrt{0^2 + (106037 + 7.4\times10^9 Q)^2}$$

$$E_q = \sqrt{0 + (106037 + 7.4\times10^9 Q)^2} \quad \text{since } E = 0$$

$$E_q = 106037 + 7.4\times10^9 Q$$

make Q subject of the formula

$$7.4\times10^9 q = 0 - 106037$$

$$7.4\times10^9 q = -106037$$

$$q = \frac{-106037}{7.4\times10^9} = 0.000014329 \simeq 1.43\times10^{-5}$$

$$q = 1.43\ \mu C$$

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$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad \text{---} \; ***$$

using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Eqn *** becomes $B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2+y^2)^{1/2}} \right]_{-a}^{a}$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

when a is larger than x

$(x^2+a^2)^{1/2} \approx a$, as $a \to \infty$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

Thus, at all points in a circle of radius r, around the conductor, the magnitude of B is, $B = \frac{\mu_0 I}{2\pi r}$

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(2a) Distinguish between the terms: Electric field & electric field intensity.

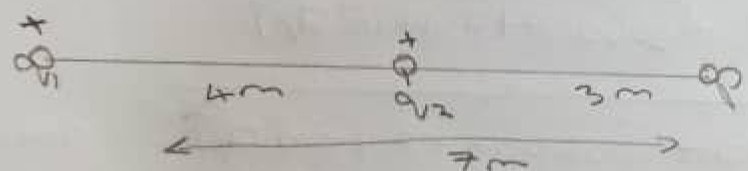
Answer

Electric field is the region or space in which an electric charge will experience an electric force. While an Electric field intensity is defined as the force per unit charge. It is measured in N/C. $F/q_0 \rightarrow N/C$

(b) A positive charge $Q_1 = 8nC$ is at the Origin, a second positive charge $Q_2 = 12nC$ is on the x-axis at $x = 4m$. ~~find~~

i The net electric field at point P ont x axis at $x = 7m$.

Soln



$$E_1 = \frac{Kq_1}{r_{q_1}^2} = \frac{9\times10^9 \times 8\times10^{-9}}{(7)^2} = 1.47\ N/C$$

$$E_2 = \frac{Kq_2}{r_{q_2}^2} = \frac{9\times10^9 \times 12\times10^{-9}}{(3)^2} = 12\ N/C$$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 \quad = 1.47 + 12 = 13.47\ N/C \approx 13.5\ N/C$$

$$1 = \frac{9\times10^9 \times (q_1 q_2 \times 5\times10^{-5})}{2^2}$$

$4 = 9\times10^9 \times 5\times10^{-5} q_1 + 9\times10^9 q_2$

$4 = 4.5\times10^5 q_1 + 9\times10^9 q_2$

Using ~~the~~ quadratic equation

~~$9\times10^9 q_2 - 4.5\times10^5 q_1 + 4 = 0$~~

$9\times10^9 q_2 + 4.5\times10^5 q_1 - 4 = 0$

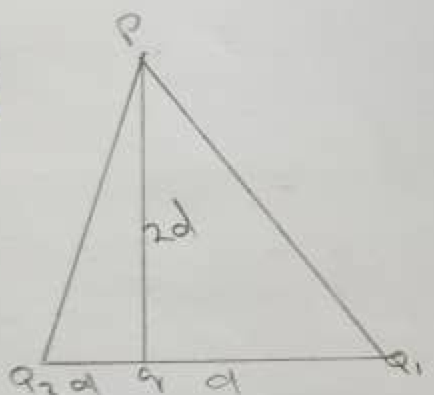
~~$q_1 = 0$~~

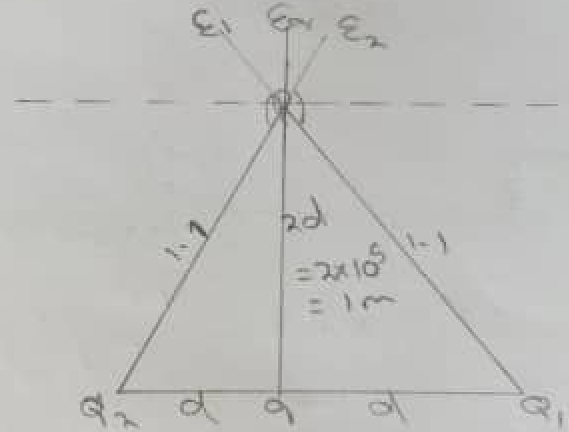
~~$q_1 = 0.00001$~~

$q_1 = -0.00005 7702 \quad \simeq q_1 = -5.8\times10^{-5}C$

$q_2 = 0.000007702 \quad \simeq q_2 = 7.7\times10^{-6}C$

1c





$Q_1 = Q_2 = 8\mu C$

$d = 0.5m$

$E_{q_1} = \frac{kq_1}{r_1^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.1)^2} = \cancel{5779.795}\ 59504\ N/C$

$E_{q_2} = \frac{kq_2}{r_2^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.1)^2} = 59504\ N/C$

$E_Q = \frac{kQ}{r^2} = \frac{9\times10^9 \times Q}{(1.1)^2} = 7.4\times10^9 Q\ N/C$

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5a) State the Biot-Savart law.

It states that.

(i) the vector $\vec{dB}$ is perpendicular both to $d\vec{l}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $d\vec{l}$ toward P.

(ii) The magnitude of $\vec{dB}$ is proportional to I, ~~where θ~~ is and the magnitude of the length of $d\vec{l}$.

(iii) The magnitude of $\vec{dB}$ is inversely proportional to r^2, where r is the distance from $d\vec{l}$ to P.

(iv) The magnitude of $\vec{dB}$ is proportional to $\sin\theta$, where θ is the angle between $\hat{r}$ and dl.

In summary

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{l} \times \hat{r}}{r^2}$$

where μ_0 is a constant called permeability of free space.

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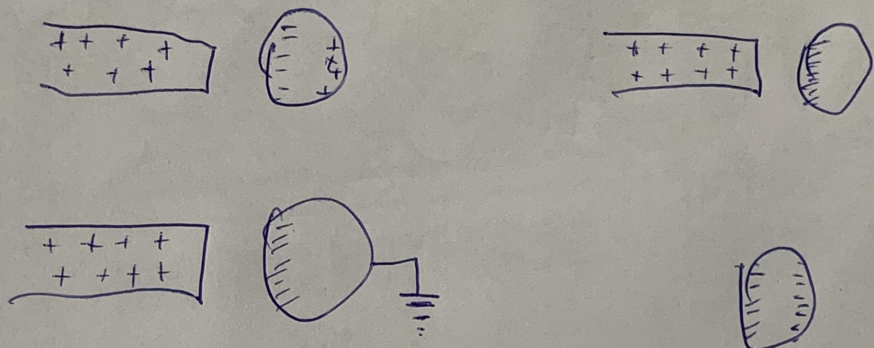
i) Charging by induction

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction.

Consider a positively charged rubber rod brought near a natural (unchanged) conducting sphere that is insulated so that there is no conducting path to ground as shown below



1b) $k = 9 \times 10^9$

$q_1 + q_2 = 5 \times 10^{-5} C$

$f = 1N$, $d = 2m$

Calculate the charge on each sphere

recall that $k = 9 \times 10^9$

$$f = \frac{k q_1 q_2}{r^2}$$