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]DEPARTMENT: COMPUTER SCIENCE
MATRIC NO: 19/SCI01/028

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ASSIGNMENT

1a. $y = \sin\left(\frac{3}{x^2}\right)$

Solution

$y = \sin\left(\frac{3}{x^2}\right) = f(x) = \sin\left(\frac{3}{x^2}\right) = f(x+\Delta x) = \sin\left(\frac{3}{(x+\Delta x)^2}\right) = \sin\left(\frac{3}{x^2+2x\Delta x+\Delta x}\right.$

$f(x+\Delta x) = \sin\left(\frac{x}{x^2+2x\Delta x+(\Delta x)^2}\right)$

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\sin\left(\frac{3}{x^2+2x\Delta x+(\Delta x)^2}\right) - \sin\left(\frac{3}{x^2}\right)}{\Delta x}$$

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{2\cos\left(\frac{\frac{3}{x^2+2x\Delta x+(\Delta x)^2} + \frac{3}{x^2}}{2}\right)\sin\left(\frac{\frac{3}{x^2+2x\Delta x+(\Delta x)^2} - \frac{3}{x^2}}{2}\right)}{\Delta x}$$

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = 2\cos\left(\frac{6x^2+6x\Delta x+3\Delta x^2}{2x^2(x^2+2x\Delta x+(\Delta x)^2)}\right)\sin\left(\frac{-6x\Delta x-3(\Delta x)^2}{2x^2(x^2+2x\Delta x+(\Delta x)^2)}\right)$$

$$= -2\cos\left(6x^2+6x\Delta x+3(\Delta x)^2\right)\sin\left(6x\Delta x+3(\Delta x)^2\right.$$

1b $y = \frac{4}{x^3}$

Solution

$f(x) = \frac{4}{x^3}$

Using $\frac{f(x+\Delta x)-f(x)}{\Delta x}$

$f(x+\Delta x) = \frac{4}{(x+\Delta x)^3}$

$$= \frac{4}{(x+\Delta x)(x+\Delta x)^2} = \frac{4}{(x+\Delta x)(x^2+2x\Delta x+(\Delta x)^2)} = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^2}$$

$$f_{\Delta x} = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$f(x+\Delta x)-f(x) = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3} - \frac{4}{x^3}$$

$$f(x+\Delta x)-f(x) = \frac{4x^3-(4[x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3])}{x^3(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$f(x+\Delta x)-f(x) = \frac{4x^3-(4[x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3])}{x^3(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$f(x+\Delta x)-f(x) = \frac{4x^3-(4x^3+12x^2\Delta x+12x(\Delta x)^2+4(\Delta x)^3)}{x^3(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^2}$$

$$f(x+\Delta x)-f(x) = \frac{\cancel{4x^3}-\cancel{4x^3}-12x^2\Delta x-12x(\Delta x)^2-4(\Delta x)^3}{x^6+3x^5\Delta x+3x^4(\Delta x)^2+x^3(\Delta x)^3}$$

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{-12x^2\Delta x-12x(\Delta x)^2-4(\Delta x)^3}{x^6+3x^5\Delta x+3x^4(\Delta x)^2+x^3(\Delta x)^3} \div \Delta x$$

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\cancel{\Delta x}(-12x^2-12x\Delta x-4(\Delta x)^2)}{x^6+3x^5\Delta x+3x^4(\Delta x)^2+x^3(\Delta x)^3} \times \frac{1}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{-12x^2-12x\Delta x-4(\Delta x)^2}{x^6+3x^5\Delta x+3x^4(\Delta x)^2+x^3(\Delta x)^3} = \frac{-12x^2-12x(0)-4(0)^2}{x^6+3x^5(0)+(3x^4(0)^2+x^3(0)^3}$$

$$\lim_{\Delta x \to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{-12x^2}{x^6} = -12x^{2-6}$$

$$= -12x^{-4} \text{ OR } \frac{-12}{x^4}$$

2a. $\dfrac{dx}{(x^2+36)}$

Solution

$$\int \frac{1}{(x^2+36)}dx = \int \frac{1}{36\left(\frac{x^2}{36}+1\right)}dx = \frac{1}{36}\int \frac{1}{\left(\frac{x^2}{36}+1\right)}dx = \frac{1}{36}\int \frac{1}{\left(\frac{x}{6}\right)^2+1}dx$$

$u = \dfrac{x}{6}$

$\dfrac{du}{dx} = \dfrac{1}{6}$

$dx = 6du$

$$= \frac{1}{36}\int \frac{1}{u^2+1}dx = \frac{1}{36}\int \frac{1}{u^2+1}\cdot 6du = \frac{\cancel{6}^1}{\cancel{36}_6}\int \frac{1}{u^2+1}du$$

Recall $\int \frac{1}{u^2+1}du$ is a standard integral, so it equates to arctan u

$= \dfrac{1}{6}\arctan u + c, \; u = \dfrac{x}{6}$

$= \dfrac{1}{6}\arctan(x/6) + c$

$= \dfrac{\arctan(x/6) + c}{6}$

2b. $\dfrac{dx}{(x^2+13)}$

Solution

$$\int \frac{1}{(x^2+13)}dx = \int \frac{1}{13\left(\frac{x^2}{13}+1\right)}dx = \frac{1}{13}\int \frac{1}{\frac{x^2}{13}+1}dx = \frac{1}{13}\int \frac{1}{\left(\frac{x}{\sqrt{13}}\right)^2+1}dx$$

$u = \dfrac{x}{\sqrt{13}}$

(Rationalize)

$$u = \frac{x}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}x}{13}$$

$$u = \frac{\sqrt{13}x}{13}$$

$$\frac{du}{dx} = \frac{\sqrt{13}}{13}$$

$$dx = \frac{13}{\sqrt{13}}\, du$$

$$= \frac{1}{13}\int \frac{1}{u^2+1}\, dx = \frac{1}{13}\int \frac{1}{u^2+1} \times \frac{13}{\sqrt{13}}\, du = \frac{13}{13\sqrt{13}} \int \frac{1}{u^2+1}\, du$$

Recall, $\int \frac{1}{u^2+1}\, du$ is a standard integral, so it equates to arctan u

$$= \frac{13}{13\sqrt{13}} \arctan u + c, \quad u = \frac{\sqrt{13}x}{13}$$

$$= \frac{\sqrt{13}}{13} \arctan\left(\sqrt{13}x/13\right) + c$$

$$= \frac{\sqrt{13} \times \arctan\left(\sqrt{13}x/13\right)}{13} + c$$