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COMPUTER ENGINEERING

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MAT 102

SERIAL NO: 35

1 A particle moves along a curve $x = 8t^3$, $y = 4t^3 - 7t$ and $z = t + 3$, where t is time. Find its

(i) velocity (ii) acceleration

Sol

$$x = 8t^3 \quad y = 4t^3 - 7t \quad z = t + 3$$

$$\text{velocity} = \frac{ds}{dt}$$

$$r = xi + yj + zk$$

$$r = (8t^3)i + (4t^3 - 7t)j + (t + 3)k$$

$$\frac{ds}{dt} = 24t^2i + (12t^2 - 7)j + k$$

$$\therefore \text{velocity } \frac{ds}{dt} = 24t^2i + (12t^2 - 7)j + k$$

$$\text{ii Acceleration } \left(\frac{d^2s}{dt^2} \right) = 48ti + 24tj$$

2 Find the unit tangent vector to the space curve $x = 3t$, $y = t^3$ and $z = t^2$ at $t = 1$

Sol

$$T = \frac{ds/dt}{|ds/dt|}$$

$$r = xi + yj + zk$$

$$r = 3ti + t^3j + t^2k$$

$$\frac{ds}{dt} = 3i + 3t^2j + 2tk$$

$$\text{At } t = 1$$

$$\frac{ds}{dt} = 3i + 3(1)^2j + 2(1)k$$

$$\therefore \frac{ds}{dt} = 3i + 3j + 2k$$

$$\left| \frac{ds}{dt} \right| = \sqrt{(3)^2 + (3)^2 + (2)^2}$$

$$\left| \frac{ds}{dt} \right| = \sqrt{9 + 9 + 4}$$

$$\left| \frac{ds}{dt} \right| = \sqrt{22}$$

$$\therefore T = \frac{3i + 3j + 2k}{\sqrt{22}}$$

$$T = \frac{3i}{\sqrt{22}} + \frac{3j}{\sqrt{22}} + \frac{2k}{\sqrt{22}}$$