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Assignment for Dr. Oyelami's group
1) find the limit of the function $((4x^2 - \sin x)/(x^3))$ as $x \to 0$

Solution

Given $\frac{4x^2 - \sin x}{x^3}$, by direct substitution as x tends to 0, we have

$$\frac{4(0) - \sin 0}{0^3}$$

$$= \frac{0-0}{0} = \frac{0}{0} = \text{undefined}$$

use L'Hopital's rule

$$\lim_{x \to 0} \left(\frac{4x^2 - \sin x}{x^3} \right) = \frac{8x - \cos x}{3x^2} = \frac{8x - \cos x}{3x^2}$$

ie by differentiating

$$\therefore \lim_{x \to 0} \left(\frac{8(0) - \cos 0}{3(0)^2} \right) = \frac{0-1}{0} = \frac{-1}{0} = \text{undefined.}$$

$\therefore$ by further differentiation

$$\frac{8x - \cos x}{3x^2} = \frac{8 - (-\sin x)}{6x} = \frac{8 + \sin x}{6x}$$

$$\therefore \lim_{x \to 0} \left(\frac{8 + \sin x}{6x} \right) = \frac{8 + \sin 0}{6(0)} = \text{undefined}$$

By further differentiation

$$\frac{8 + \sin x}{6x} = \frac{\cos x}{6}$$

$$\lim_{x \to 0} = \left(\frac{\cos 0}{6}\right) = \frac{1}{6}$$

2) If $y = (7x^2 \cos 8x)/e^{3x}$, find the derivative of y with respect to x.

Given $y = \dfrac{7x^2 \cos 8x}{e^{3x}}$

Let $u = 7x^2$, $\frac{du}{dx} = 14x$

$v = \cos 8x$, $\frac{dv}{dx} = -8\sin 8x$

$w = e^{3x} = \frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left(\frac{1}{u} \times \frac{du}{dx} + \frac{1}{v} \times \frac{dv}{dx} - \frac{1}{w} \times \frac{dw}{dx}\right)$$

$$\frac{dy}{dx} = y\left(\frac{1}{7x^2} \times 14x + \frac{1}{\cos 8x} \times 8 \cdot \sin 8x \cdot \frac{1}{e^{3x}} \times 3e^{3x}\right.$$

$$= y\left(\frac{2}{x} - \frac{8\sin 8x}{\cos 8x} - 3\right)$$

$$= y\left(\frac{2}{x} - 8\tan 8x - 3\right)$$

3) If $y = \cos(5x^2 + 6x)$. find $\frac{dy}{dx}$

Solution

Given $y = \cos(5x^2 + 6x)$

let $u = 5x^2 + 6x$

$\therefore \frac{du}{dx} = 10x + 6$

$\therefore y = \cos u$

$\frac{dy}{du} = -\sin u$

$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{dy}{du} = 10x + 6 \times -\sin u$

$\frac{dy}{du} = -\sin u \times 10x + 6$ but $u = 5x^2 + 6x$

$\frac{dy}{dx} = -\sin u\, 5x^2 + 6x(10x + 6)$,,

4) find the integral of the following

a) $\frac{3}{(4x+1)}\, dx$

Solution

$\int \frac{3}{4x+1}\, dx$, let $u = 4x + 1$, $\frac{du}{dx} = \frac{4}{1}$

$\therefore dx = \frac{du}{4}$ but $x = \frac{u-1}{4}$

$\therefore \int \frac{du}{4} \times 3 \times \frac{1}{4x+1} = \int \frac{3\,du}{4(4x+1)}$

$= \frac{3}{4}\int \frac{du}{4x+1} = \frac{3}{4}\int \frac{du}{u} = \frac{3}{4_2} \times \frac{x}{u^2} + c$

by final simplification $= \frac{3}{2u^2} + c \quad = \frac{3}{2(4x+1)} + c$

b) $\frac{dx}{(x^2+49)}$

Solution

$$\int \frac{dx}{(x^2+49)} = \int \frac{dx}{x^2+7^2}$$

$x = 7\tan\theta$

$\frac{dx}{d\theta} = 7\sec^2\theta \quad dx = 7\sec^2\theta\, d\theta$

$= x^2+7^2 = 7\tan^2\theta + 7^2 = 7^2(\tan^2\theta+1)$

factorizing after substituting.

$$\int \frac{7\sec^2\theta\, d\theta}{7^2\sec^2\theta} = \int \frac{d\theta}{7} = \frac{1}{7}\int d\theta = \frac{1}{7}\theta + c$$

but $\theta = \tan^{-1}\frac{x}{7}$

$= \frac{1}{7}\tan^{-1}\frac{x}{7} + c$

$(e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

Solution

$\int e^{6x} + 9x - \sin 7x + \cos 8x\, dx$

$= \frac{1}{6}e^{6x} + \frac{9x^2}{2} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + c$