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Answers

1) If $A = 3i + 7j - 2k$, $B = i + 3j + 7k$, $C = 9i - 4j + 6k$. find the angle between.

i) A and C.

Solution

Given $\bar{A} = 3i + 7j - 2k$, and $\bar{B} = i + 3j + 7k$

$\hat{A} \cdot \bar{B} = (3i + 7j - 2k) \cdot (i + 3j + 7k)$

$= 3 + 27 - 14 = 16$

$\cos\theta = \dfrac{\bar{A} \cdot \bar{B}}{|A||B|}$

$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{9 + 49 + 4} = \sqrt{62}$

$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{1 + 9 + 49} = \sqrt{59}$

$\cos\theta = \dfrac{A \cdot B}{|A||B|} = \dfrac{16}{\sqrt{62} \times \sqrt{59}} = \dfrac{16}{60.48} = 0.2645$

$\theta = \cos^{-1} 0.2645 = 74.66°$ $\therefore 74.66°$ is the $\angle$ between them.

ii B and C

Solution

Given $\bar{B} = i + 3j + 7k$ and $\bar{C} = 9i - 4j + 6k$

$\bar{B} \cdot \bar{C} = (i + 3j + 7k) \cdot (9i - 4j + 6k)$

$= 9 - 12 + 42 = 39$

$\cos\theta = \dfrac{\bar{B} \cdot \bar{C}}{|B||C|}$

$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{1 + 9 + 49} = \sqrt{59}$

$|C| = \sqrt{9^2+(-4)^2+6^2} = \sqrt{81+16+36} = \sqrt{133}$

$\theta = \cos^{-1} \frac{\bar{B}\cdot C}{|B||C|} = \cos^{-1} \frac{39}{\sqrt{59}\times\sqrt{133}} = \cos^{-1} 0.44026$

$\theta = 63.88°$ $\therefore$ $63.88°$ is the angle between $\bar{B}$ and $\bar{C}$

ii The unit vector in the direction of $(\bar{A}+\bar{B}+\bar{C})$

Solution

Given $\bar{A} = 3i+7j-2k$, $\bar{B} = i+3j+7k$, $\bar{C} = 9i-4j+6k$

$(\bar{A}+\bar{B}+\bar{C}) = (3i+7j-2k)+(i+3j+7k)+(9i-4j+6k)$

$= 13i+6j+11k$

$|A+B+C| = \sqrt{13^2+6^2+11^2} = \sqrt{169+36+121} = \sqrt{326}$

$= 18.06$

$e_{A+B+C} = \frac{13i+6j+11k}{18.06} = \frac{13}{18.06}i + \frac{6j}{18.06} + \frac{11k}{18.06}$

2) A particle moves along a curve, $x = 8t^2$, $y = t^2-4t$, $z = t+1$ where t is time. find the modulus of acceleration at $t=1$

Soln

When $x = 8t^2$, $y = t^2-4t$, $z = t+1$

Let $r = 8t^2 i + (t^2-4t)j + (t+1)k$

Since t = time

velocity, $\frac{dr}{dt} = 16t\,i + (2t-4)j + k$

Accelration $= \frac{d^2r}{dt^2} = 16i+2j$, $\left|\frac{d^2r}{dt^2}\right|_{t=1} = \sqrt{16^2+2^2}$

$\therefore$ The modulus of its acceleration at $t=1$ = $\sqrt{256+4} = \sqrt{260} = 2\sqrt{65}$

3) If $A = 4i + 2j - 4k$, $B = 8i - 2j + k$, $C = i + 4j - 3k$
find the vector triple product $(A \times B) \times C$

Solution

$A = 4i + 2j - 4k$, $B = 8i - 2j + k$, $C = i + 4j - 3k$

to find the vector triple product
First find $A \times B$

$$(A \times B) = \begin{vmatrix} + & - & + \\ i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & -4 \\ -2 & 1 \end{vmatrix} - j\begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k\begin{vmatrix} 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$= i(2 - 8) - j(4 + 32) + k(-8 - 16)$$

$$= i(-6) - j(36) + k(-24)$$

$$A \times B = -6i - 36j - 24k$$

$$\therefore (A \times B) \times C = \begin{vmatrix} + & - & + \\ i & j & k \\ -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$(A \times B) \times C = i\begin{vmatrix} -36 & -24 \\ 4 & -3 \end{vmatrix} - j\begin{vmatrix} -6 & -24 \\ 1 & -3 \end{vmatrix} + k\begin{vmatrix} -6 & -36 \\ 1 & 4 \end{vmatrix}$$

$$= i(108 + 96) - j(18 + 24) + k(-24 + 36)$$

$$= i(204) - j(42) + k(12)$$

$$= 204i - 42j + 12k$$