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MECHATRONICS ENGINEERING 19/ENG05/060

MAT104

(1) $\lim_{x \to 0} = \frac{4x^2 - \sin x}{x^3}$

$\frac{dy}{dx} = \frac{8x - \cos x}{3x^2}$

$\frac{d^2y}{dx^2} = \frac{8 + \sin x}{6x}$

$\frac{d^3y}{dx^3} = \frac{0 + \cos x}{6}$

$\lim_{x \to 0} = \frac{\cos 0}{6} = \frac{1}{6}$

(2) If $y = \frac{(7x^2 (\cos 8x))}{e^{3x}}$

$\frac{dy}{dx} = y\left(\frac{1}{7x^2} \times 14x + \frac{1}{\cos 8x} \times (-8 \sin 8x) - \frac{1}{e^{3x}} \times 3e^{3x}\right)$

$\frac{dy}{dx} = y\,(2x - 8\tan 8x - 3)$

$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}}\,(2x - 8\tan 8x - 3)$

(3) If $y = \cos(5x^2 + 6x)$, find dy/dx

let $u = 5x^2 + 6x$

$\frac{du}{dx} = 10x + 6$

$y = \log u$

$\frac{dy}{du} = -\sin u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -\sin u \times (10x + 6)$

$\frac{dy}{dx} = -10x \sin u - 6 \sin u$

$\frac{dy}{dx} = -10x \sin(5x^2 + 6x) - 6 \sin(5x^2 + 6x)$

(4) $\int \frac{3\,dx}{4x+1}\,dx$

$3\int \frac{1}{4x+1}\,dx$

let $u = 4x + 1$

$\frac{du}{dx} = 4$

$dx = \frac{du}{4}$

$$3\int \frac{1}{u} \cdot \frac{du}{4}$$

$$\frac{3}{4}\int \frac{du}{4}$$

$$\frac{3}{4}\int u^{-1}\,du$$

$$\frac{3}{4}\left[\frac{u^{-2}}{-2}\right] + C$$

$$\frac{-3}{8} u^{-2} + C$$

$$= \frac{-3}{8}(4x+1) + C$$

(b) $$\int \frac{dx}{x^2+49} = \int \frac{dx}{x^2+7^2}$$

Let $x = 7\tan\theta$

$$\frac{dx}{d\theta} = 7\sec^2\theta$$

$$dx = 7\sec^2\theta\, d\theta$$

$$x^2+7^2 = 7^2 + 7^2\tan\theta$$

$$= 7^2(1+\tan\theta)$$

$= 7^2 \sec^2\theta$

$$\int \frac{7\sec^2\theta \, d\theta}{7^2 \sec^2\theta}$$

$$= \frac{1}{7} \int d\theta$$

$$= \frac{1}{7} \tan^{-1} \frac{x}{7}$$

(c) $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\, dx$

$$\frac{1}{6} e^{6x} + \frac{9x^4}{4} + \frac{1}{7} \cos 7x + \frac{1}{8} \sin 8x + C$$

$$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{\cos 7x}{7} + \frac{\sin 8x}{8} + C$$

(d) $\int x \sqrt{9 + x^2}\, dx$

$u = 9 + x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\int x\, u^{1/2} \cdot \frac{du}{2x}$

$$\frac{1}{2}\int u^{1/2}\,du$$

$$\frac{1}{2}\left|\frac{u^{3/2}}{3/2}\right|+C$$

$$\frac{1}{2}\times\frac{2u^{3/2}}{3}+C$$

$$\frac{2u^{3/2}}{6}+C$$

$$\frac{1}{3}(9+x^2)^{3/2}+C$$