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MAT 102  
19 / ENG 05 / 052

Mechatronics Engineering

(i) If  $A = 3i + 7j - 2k$ ,  $B = i + 3j + 7k$ ,  $C = 9i - 4j + 6k$ , Find the angle between

(i) A and C

(ii) B and C

(iii) The unit vector in the direction of  $(A+B+C)$

solution

$$\cos \theta = \frac{A \cdot C}{|A| |C|}$$

$$\cos \theta = \frac{27 - 28 - 12}{\sqrt{9 + 49 + 4} \cdot \sqrt{81 + 16 + 36}}$$

$$\cos \theta = \frac{-13}{\sqrt{62} \cdot \sqrt{133}}$$

$$\cos \theta = \frac{-13}{\sqrt{62} \cdot \sqrt{133}}$$

$$\cos \theta = \frac{-13}{90.81}$$

$$\theta = \cos^{-1} - 0.1432$$

$$\theta = 98.7^\circ$$

$$(ii) \cos \theta = \frac{B \cdot C}{|B| \cdot |C|}$$

$$\cos \theta = \frac{9 - 12 + 42}{\sqrt{1+9+49} \cdot \sqrt{81+16+36}}$$

$$\cos \theta = \frac{39}{\sqrt{59} \cdot \sqrt{133}}$$

$$\cos \theta = \frac{39}{88.58}$$

$$\cos \theta = 0.4403$$

$$\theta = \cos^{-1} 0.4403$$

$$\theta = 63.88^\circ$$

$$(iii) e(A+B+C)$$

$$A+B+C = (3+1+9)i + (7+3-4)j + (-2-4+7)k$$

$$= 13i + 6j + 1k$$

$$e(A+B+C) = \frac{(\bar{A} + \bar{B} + \bar{C})}{|A+B+C|}$$

$$e(A+B+C) = \frac{13i + 6j + 1k}{\sqrt{169 + 36 + 1}} = \frac{13i + 6j + 1k}{\sqrt{206}}$$

(2) A particle moves along a curve,  $x = -8t^2$ ,  $y = t^2$ ,  $z = t + 1$ , where  $t$  is time. Find the modulus of acceleration at  $t = 1$

$$r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$r = -8t^2\mathbf{i} + t^2\mathbf{j} + (t+1)\mathbf{k}$$

$$\frac{dr}{dt} = -16t\mathbf{i} + 2t\mathbf{j} + \mathbf{k}$$

$$\left| \frac{d^2r}{dt^2} \right|_{t=1} = -16\mathbf{i} + 2\mathbf{j}$$

$$= \sqrt{256 + 4}$$

$$= \sqrt{260}$$

$$= 2\sqrt{65}$$

(3) If  $A = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ ,  $B = 8\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ ,  $C = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$  find the vector triple product  $(A \times B) \times C$

$$A \times B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$= \mathbf{i} \begin{vmatrix} 2 & -4 \\ -2 & 1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$= \mathbf{i}(2 - 8) - \mathbf{j}(4 + 32) + \mathbf{k}(-8 + 16)$$

$$= -6\mathbf{i} - 36\mathbf{j} + 8\mathbf{k}$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & 8 \\ 4 & 4 & -3 \end{vmatrix}$$

$$= i \begin{vmatrix} -36 & 8 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} -6 & 8 \\ 4 & -3 \end{vmatrix} + k \begin{vmatrix} -6 & -36 \\ 4 & 4 \end{vmatrix}$$

$$= i (108 - 32) - j (18 - 8) + k (-24 + 36)$$

$$= 72i - 10j + 12k$$