NAME: Ike-Egbuonu Marvellous Chinelo
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DR. OYELAMI

1. Find the ~~Integral~~ limit of the function $\left\{\frac{4x^2 - \sin x}{x^3}\right\}$ as $x \to 0$

$$\left\{\frac{4x^2-\sin x}{x^3}\right\}_{x\to 0} = \left\{\frac{8x-\cos x}{3x^2}\right\} = \left\{\frac{8+\sin x}{6x}\right\} = \left\{\frac{\cos x}{6}\right\}$$

$$= \frac{\cos 0}{6} = \frac{1}{6}$$

2. If $y = \frac{(7x^2 \cos 8x)}{e^{3x}}$, find the derivative.

$$y = \frac{uv}{w}$$

$$\frac{dy}{dx} = y\left[\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}\right]$$

$u = 7x^2$, $v = \cos 8x$, $w = e^{3x}$

$\frac{du}{dx} = 14x$, $\frac{dv}{dx} = -8\sin 8x$, $\frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{14x}{7x^2} - \frac{8\sin 8x}{\cos 8x} - \frac{3e^{3x}}{e^{3x}}\right]$$

$$\frac{dy}{dx} = \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{2}{x} - 8\tan 8x - 3\right]$$

3. $y = \cos(5x^2 + 6x)$, find the derivative.

Let $u = 5x^2 + 6x$, $\frac{du}{dx} = 10x + 6$

$y = \cos u$, $\frac{dy}{du} = -\sin u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\sin u \times (10x+6)$$

$$= (-\sin(5x^2+6x))(10x+6)$$

4d) $\int x(\sqrt{(9+x^2)})\,dx$

Let $u = 9+x^2$, $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$

$$\int x(\sqrt{(9+x^2)}) = \int x(\sqrt{u})\frac{du}{2x}$$

$$= \frac{1}{2}\int(\sqrt{u})\,du$$

$$= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\left[\frac{u^{3/2}}{\frac{3}{2}}\right] + C$$

$$= \frac{1}{3}\left[(9+x^2)^{3/2}\right] + C$$

4a. Find the Integral of $\int \frac{3\,dx}{(4x+1)}$

Let $u = 4x+1$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$dx = \frac{du}{4}$

$$\int \frac{3\,dx}{(4x+1)} = \int \frac{\frac{3du}{4}}{u} = \frac{3}{4}\int\frac{du}{u} = \frac{3}{4}\int\frac{1}{u}$$

$$= \frac{3}{4}\ln(u) + c$$

$$= \frac{3}{4}\ln(4x+1) + c = \frac{3\ln(4x+1)}{4} + c$$

b. $\int \frac{dx}{x^2+49} = \int\frac{dx}{x^2+7^2}$

$x = 7\tan\theta$, $\theta = \tan^{-1}\frac{x}{7}$

$\frac{dx}{d\theta} = 7\sec^2\theta$

$dx = 7\sec^2\theta\,d\theta$ --- (i)

$x^2+7^2 = 7^2 + 7^2\tan^2\theta = 7^2(1+\tan^2\theta)$

$= 7^2\sec^2\theta$ --- (ii)

$$\int\frac{dx}{x^2+7^2} = \int\frac{7\sec^2\theta\,d\theta}{7^2\sec^2\theta} = \int\frac{d\theta}{7} = \frac{1}{7}\int d\theta$$

$$= \frac{1}{7}[\theta] + c = \frac{1}{7}\tan^{-1}\frac{x}{7} + C$$

c. $\int(e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

$$= \left\{\frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin x\right\} + c$$