Oyelaye Numdeen Opeyemi
Mechatronics Engineering MAT 104
19/ENG05/058

$\frac{4x^2 - \sin x}{x^3}$ as $x \to 0$

$y = (7x^2 \cos 8x)/e^{3x}$

$= 7\left(\frac{dy}{dx} \frac{x^2 \cos 8x}{e^{3x}}\right)$

Use quotient rule to find the derivative of $\frac{x \cos 8x}{e^{3x}}$

$$7 \times \frac{e^{3x}\left(\frac{dy}{dx} x^2 \cos 8x\right) - x^2 \cos 8x \left(\frac{dy}{dx} e^{3x}\right)}{e^{6x}}$$

Use product rule to find the derivative of x^2 $\cos 8x$. The product rule states that $(fg)' = f'g + fg'$

$$7 \times \frac{e^{3x}\left[\left(\frac{dy}{dx} x^2\right) \cos 8x + x^2 \left(\frac{dy}{dx} \cos 8x\right)\right] - x^2 \cos 8x \left(\frac{dy}{dx} e^{3x}\right)}{e^{6x}}$$

Use power rule

$$7 \times \frac{e^{3x}\left(2x \cos 8x + x^2 \left(\frac{dy}{dx} \cos 8x\right)\right) - x^2 \cos 8x \left(\frac{dy}{dx} e^{3x}\right)}{e^{6x}}$$

Use chain rule on $\frac{dy}{dx} \cos 8x$. Let $u = 8x$. Use trigonometric differentiation the derivative of $\cos u$ is $-\sin u$

$$7 \times \frac{e^{3x}\left(2x \cos 8x - x^2 \sin 8x \left(\frac{dy}{dx} 8x\right)\right) - x^2 \cos 8x \left(\frac{dy}{dx} e^{3x}\right)}{e^{6x}}$$

Use power rule.

$$7 \times \frac{e^{3x}\left(2x \cos 8x - 8x^2 \sin 8x\right) - x^2 \cos 8x \left(\frac{dy}{dx} e^{3x}\right)}{e^{6x}}$$

Use chain rule on $\frac{dy}{dx} e^{3x}$

$$7 \times \frac{e^{3x}\left(2x \cos 8x - 8x^2 \sin 8x\right) - x^2 \cos 8x \, e^{3x} \left(\frac{dy}{dx} 3x\right)}{e^{6x}}$$

Use power rule! $\frac{dy}{dx} = x^n = nx^{n-1}$

$$7 \times \frac{e^{3x}\left(2x \cos 8x - 8x^2 \sin 8x\right) - 3x^2 e^{3x} \cos 8x}{e^{6x}}$$

3 $y = \cos(5x^2 + 6x)$

use chain rule of $\frac{dy}{dx}\cos(5x^2+6x)$

Let $u = 5x^2 + 6x$. use trigonometric differ

The derivative of $\cos u$ is $-\sin u$

$-\sin(5x^2+6x)\left(\frac{dy}{dx}5x^2+6x\right)$

Using power rule for $\frac{dy}{dx}x^n = nx^{n-1}$

$= -(10x+6)\sin(5x^2+6x)$

4a $\int \frac{3dx}{4x+1}\,dx$

$3d\int \frac{x}{4x+1}\,dx$

Polynomal division

$3d\int \frac{1}{4} - \frac{1}{4(4x+1)}\,dx$

use sum rule

$3d\left(\int \frac{1}{4}\,dx - \int \frac{1}{4(4x+1)}\,dx\right)$

$3d\left(\frac{x}{4} - \int \frac{1}{4(4x+1)}\right)dx$

$3d\left(\frac{x}{4} - \frac{1}{4}\int \frac{1}{4x+1}\,dx\right)$

Let $u = 4x+1$, $du = 4dx$, then $dx = \frac{1}{4}du$

$= \int \frac{1}{4u}\,du$

$\frac{1}{4}\int \frac{1}{u}\,du = \frac{\ln u}{4}$

$\frac{\ln(4x+1)}{4}$

$$3d\left(\frac{x}{4} - \frac{\ln(4x+1)}{16}\right)$$

$$3d\left(\frac{x}{4} - \frac{\ln(4x+1)}{16}\right) + C$$

b $\frac{dx}{x^2+49} = \int \frac{1}{x^2+49}\,dx$

Let $x = 7\tan u$, $du = 7\sec^2 u\,du$

$$\int \frac{1}{(7\tan u)^2 + 49} \times 7\sec^2 u\,du$$

Simplify $\int \frac{1}{7}\,du = \frac{u}{7}$

$$u = \tan^{-1}\left(\frac{1}{7}x\right)$$

$$\frac{\tan^{-1}\left(\frac{1}{7}x\right)}{7} = \frac{\tan^{-1}\left(\frac{1}{7}x\right)}{7} + C$$

d. $\int x\sqrt{9+x^2}\,dx$

Let $u = 9+x^2$, $du = 2x\,dx$, then $xdx = \frac{1}{2}du$

$$\int \frac{\sqrt{u}}{2} = \frac{1}{2}\int \sqrt{u}\,du$$

$$= \frac{1}{2}\int \sqrt{u}\,du = \frac{u^{\frac{3}{2}}}{3}$$

$$\frac{(9+x^2)^{3/2}}{3} = \frac{(9+x^2)^{\frac{3}{2}}}{3} + C$$

Scanned by TapScanner