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DEPARTMENT: MECHATRONICS.

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ASSIGNMENT

1. find the limit of the function $\frac{(4x^2 - \sin x)}{x^3}$ as $x \to 0$.

Solution

$$\lim_{x \to 0} \left[\frac{4x^2 - \sin x}{x^3} \right]$$

$$= \lim_{x \to 0} \left[\frac{8x - \cos x}{3x^2} \right]$$

$$\lim_{x \to 0} \left[\frac{8x - (-\sin x}{6x} \right]$$

$$\lim_{x \to 0} \left[\frac{8 + \sin x}{6x} \right]$$

$$\lim_{x \to 0} \left[\frac{0 + \cos x}{6} \right]$$

$$= \frac{\cos x}{6} = \frac{\cos(0)}{6} = \frac{1}{6}$$

2. If $y = \frac{(7x^2 \cos 8x)}{e^{3x}}$, find the derivative of y with respect to x

Solution

$$y = \frac{7x^2 \cos 8x}{e^{3x}}$$

$u = 7x^2 \qquad v = \cos 8x \qquad w = e^{3x}$

$\frac{du}{dx} = 14x \qquad \frac{dv}{dx} = -8\sin 8x \qquad \frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left[\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}\right]$$

$$y = \left[\frac{1}{7x^2}[14x] + \frac{1}{\cos 8x}[-8\sin 8x] - \frac{1}{e^{3x}}[3e^{3x}]\right]$$

$$y = \left[\frac{1}{x} + (-8\tan 8x) - 1\right]$$

$$y = \left[\frac{1}{x} - 8\tan 8x - 1\right]$$

$$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}}\left[\frac{1}{x} - 8\tan 8x - 1\right]$$

2. If $y = \cos(5x^2+6x)$, find $\frac{dy}{dx}$

<u>Solution</u>

$y = \cos(5x^2+6x)$

let $u = 5x^2+6x$

$y = \cos u$

$\frac{du}{dx} = 10x+6 \qquad \frac{dy}{du} = -\sin u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$10x+6(-\sin u)$

$= 10x+6 \sin(-5x^2+6x)$

$= -10x+6 \sin(5x^2+6x)$

4. Find the integral of the following

a) $\frac{3dx}{4x+1}$

solution

$\int \frac{3dx}{4x+1} = 3\int \frac{1}{4x+1} dx$

$u = 4x+1 \quad \frac{du}{dx} = 4 \quad dx = \frac{1}{4} du$

$\frac{3}{4}\int \frac{1}{u} du$

$\frac{3}{4} \ln u$

$\frac{3}{4} \ln(4x+1) + c$

6. $\dfrac{dx}{x^2+49}$

$$\int \frac{dx}{x^2+49} = \frac{dx}{x^2+7^2}$$

$$x = 7\tan\theta$$

$$\frac{dx}{d\theta} = 7\sec^2\theta$$

$$dx = 7\sec^2\theta \, d\theta$$

$$x^2+7^2 = 7^2\tan^2\theta + 7^2$$

$$= 7^2(\tan^2\theta + 1)$$

$$= 49\sec^2\theta$$

$$= \int \frac{7\sec^2\theta \, d\theta}{49\sec^2\theta} = \int \frac{d\theta}{7} = \frac{1}{7}$$

$$= \frac{1}{7}[\theta] + c$$

$$= \frac{1}{7}\tan^{-1}\frac{x}{7} + c.$$

c. $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

$= \int e^{6x} + \int 9x^3 - \int \sin 7x + \int \cos 8x$

$= \left[\frac{1}{6}e^{6x} + \frac{9x^{3+1}}{3+1} - \left[\frac{-\cos 7x}{7}\right] + \frac{\sin 8x}{8} + c\right.$

$= \left[\frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{\cos 7x}{7} + \frac{\sin 8x}{8} + c\right.$

d. $\int x\sqrt{x^2+9}\,dx$

$u = x^2-9$

$\frac{du}{dx} = 2x \quad dx = \frac{du}{2x}$

$\frac{1}{2}\int \sqrt{u}\,du$

$= \frac{1}{2}\left[\frac{2u^{3/2}}{3}\right] du$

$= \frac{u^{3/2}}{3}$

$\frac{(x^2-9)^{3/2} + c}{3}$