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MAT 104

Assignment

1. Find the limit of the function $\{(4x^2 - \sin x)/x^3\}$ as $x \to 0$

Solution

$$\left\{\frac{4x^2 - \sin x}{x^3}\right\} \quad x \to 0$$

Using L'Hopital's Rule $\lim_{x \to a} \frac{f^n(x)}{g^n(x)}$

$$= \lim_{x \to 0}\left\{\frac{8x - \cos x}{3x^2}\right\} = \lim_{x \to 0}\left\{\frac{8 + \sin x}{6x}\right\}$$

$$= \lim_{x \to 0}\left\{\frac{\cos x}{6}\right\}$$

$$= \frac{1}{6}$$

2. If $y = (7x^2 \cos 8x)/e^{3x}$, find the derivative of y w.r.t x

Solution

$$y = \frac{7x^2 \cos 8x}{e^{3x}}$$

$$\frac{dy}{dx} = y\left[\frac{1}{u}\cdot\frac{du}{dx} + \frac{1}{v}\cdot\frac{dv}{dx} - \frac{1}{w}\cdot\frac{dw}{dx}\right]$$

$u = 7x^2, \quad v = \cos 8x, \quad w = e^{3x}$

$$\frac{du}{dx} = 14x, \quad \frac{dv}{dx} = -8\sin 8x, \quad \frac{dw}{dx} = 3e^{3x}$$

$$= y\left[\frac{1}{7x^2}\cdot 14x + \frac{1}{\cos 8x}\cdot -8\sin 8x - \frac{1}{e^{3x}}\cdot 3e^{3x}\right]$$

$$= y\left[\frac{14x}{7x^2} - \frac{8\sin 8x}{\cos 8x} - \frac{3e^{3x}}{e^{3x}}\right]$$

$$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}}\left[\frac{2}{x} - 8\tan 8x - 3\right]$$

3. If $y = \cos(5x^2 + 6x)$, find dy/dx

Solution

$$y = \cos(5x^2 + 6x)$$

$u = 5x^2 + 6x \quad ; \quad y = \cos u$

$$\frac{dy}{dx} = 10x + 6$$

$$\frac{dy}{du} = -\sin u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\sin u \cdot 10x + 6$$

$$= (-\sin(5x^2+6x)) \cdot 10x + 6$$

$$= -10x\sin(5x^2+6x) - 6\sin(5x^2+6x)$$

4\. Find the integral of the following

Solution

a. $$\int \frac{3}{(4x+1)}\,dx$$

$$u = 4x + 1 \qquad \int \frac{3}{u}\,dx$$

$$\frac{du}{dx} = 4$$

$$dx = \frac{du}{4}$$

$$= \int \frac{3}{u} \cdot \frac{du}{4}$$

$$= \frac{1}{4}\int \frac{3}{u}\,du$$

$$= \frac{1}{4}\, 3\ln u$$

$$= \frac{1}{4}\, 3\ln(4x+1)$$

$$= \frac{3}{4}\ln(4x+1) + c$$

b. $\int \frac{dx}{x^2+49} = \int \frac{dx}{x^2+7^2}$

$x = 7\tan\theta$

$\frac{dx}{d\theta} = 7\sec^2\theta$

$dx = 7\sec^2\theta\, d\theta$

$x^2+7^2 = 7^2 + 7^2\tan^2\theta$

$7^2 \cdot (\tan^2\theta + 1) = 49\sec^2\theta$

$\int \frac{7\sec^2\theta\, d\theta}{49\sec^2\theta} = \frac{1}{7}\int d\theta$

$\frac{1}{7}[\theta] + c$

$= \frac{1}{7}\tan^{-1}\frac{x}{7} + c$

c. $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

$= \frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + c$

d. $\int x\sqrt{9+x^2}\,dx$

$u = 9+x^2$ $\quad\quad\quad\quad \int x\,u^{1/2}\,dx$

$\frac{du}{dx} = 2x \;;\; dx = \frac{du}{2x}$

$\int x\,u^{1/2}\,\frac{du}{2x}$

$\frac{x}{2x}\int u^{1/2}\,du$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2}$

$= \frac{1}{2}\cdot\frac{2u^{3/2}}{3}$

$= \frac{2u^{3/2}}{6}$

$= \frac{1}{3}u^{3/2}$

$= \frac{1}{3}(9+x^2)^{3/2} + C$