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Assignment 8

$$\int \frac{1}{a^2 + x^2} dx$$
$$= \int \frac{1}{a^2 + a^2 \tan^2 \theta} \cdot a \sec^2 \theta d\theta$$
$$= \int \frac{1}{a^2 (1 + \tan^2 \theta)} \cdot a \sec^2 \theta d\theta$$
$$= \int \frac{1}{a^2 \sec^2 \theta} \cdot a \sec^2 \theta d\theta$$
$$= \int \frac{1}{a} d\theta$$
$$= \frac{1}{a} \theta + C$$
$$= \frac{1}{a} \arctan \left(\frac{x}{a} \right) + C$$

$$\int \frac{1}{a^2 - x^2} dx$$
$$= \int \frac{1}{(a-x)(a+x)} dx$$
$$= \int \frac{A}{a-x} + \frac{B}{a+x} dx$$
$$\Rightarrow \frac{1}{(a-x)(a+x)} = \frac{A}{a-x} + \frac{B}{a+x}$$
$$\Rightarrow \frac{1}{(a-x)(a+x)} = \frac{A(a+x) + B(a-x)}{(a-x)(a+x)}$$
$$\Rightarrow 1 = A(a+x) + B(a-x)$$
$$\Rightarrow 1 = Aa + Ax + Ba - Bx$$
$$\Rightarrow 1 = (A+B)a + (A-B)x$$
$$\Rightarrow \begin{cases} A+B = \frac{1}{a} \\ A-B = 0 \end{cases}$$
$$\Rightarrow \begin{cases} A = \frac{1}{2a} \\ B = \frac{1}{2a} \end{cases}$$
$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \int \frac{1}{a-x} + \frac{1}{a+x} dx$$
$$= \frac{1}{2a} \left(-\ln|a-x| + \ln|a+x| \right) + C$$
$$= \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\int \frac{1}{a^2 - x^2} dx$$
$$= \int \frac{1}{a^2 - a^2 \tan^2 \theta} \cdot a \sec^2 \theta d\theta$$
$$= \int \frac{1}{a^2 (1 - \tan^2 \theta)} \cdot a \sec^2 \theta d\theta$$
$$= \int \frac{1}{a^2 \frac{\cos^2 \theta}{\cos^2 \theta} (1 - \frac{\sin^2 \theta}{\cos^2 \theta})} \cdot a \frac{1}{\cos^2 \theta} d\theta$$
$$= \int \frac{1}{a^2 \frac{1 - \sin^2 \theta}{\cos^2 \theta}} \cdot a \frac{1}{\cos^2 \theta} d\theta$$
$$= \int \frac{1}{a^2 \cos^2 \theta} \cdot a \frac{1}{\cos^2 \theta} d\theta$$
$$= \int \frac{1}{a \cos^4 \theta} d\theta$$
$$= \frac{1}{a} \int \sec^4 \theta d\theta$$
$$= \frac{1}{a} \int \sec^2 \theta \cdot \sec^2 \theta d\theta$$
$$= \frac{1}{a} \int \sec^2 \theta \cdot \tan \theta d(\tan \theta)$$
$$= \frac{1}{a} \int \tan \theta d(\tan \theta)$$
$$= \frac{1}{a} \cdot \frac{1}{2} \tan^2 \theta + C$$
$$= \frac{1}{2a} \tan^2 \theta + C$$
$$= \frac{1}{2a} \frac{x^2}{a^2 - x^2} + C$$
$$= \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$