Oputa Clinton Chukwunwikem

19/ENG09/019

Aeronautical and Astronautical Engineering

MAT104

General Mathematics II

Answers:-

(1) If $A = 3i + 7j - 2k$, $B = i + 3j + 7k$, $C = 9i - 4j + 6k$, find the angle between:

(i) A and C

solution

Given $\bar{A} = 3i + 7j - 2k$ and $\bar{B} = i + 3j + 7k$

$$\bar{A} \cdot \bar{B} = (3i + 7j - 2k) \cdot (i + 3j + 7k)$$

$$= 3 + 27 - 14 = \underline{\underline{16}}$$

$$\cos\theta = \frac{\bar{A} \cdot \bar{B}}{|A||B|}$$

$$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{62}$$

$$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$$

$$\cos\theta = \frac{A \cdot B}{|A||B|} = \frac{16}{\sqrt{62} \times \sqrt{59}} = \frac{16}{60.48} = 0.2645$$

$$\theta = \cos^{-1} 0.2645 = 74.66^\circ$$ ans

(ii) B and C

solution

Given $\bar{B} = i + 3j + 7k$ and $\bar{C} = 9i - 4j + 6k$

$$\bar{B} \cdot \bar{C} = (i + 3j + 7k) \cdot (9i - 4j + 6k)$$

$$= 9 - 12 + 42 = 39.$$

$$\cos\theta = \frac{\bar{B} \cdot \bar{C}}{|B||C|}$$

$$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$\theta = \cos^{-1} \frac{37}{\sqrt{59} \times \sqrt{133}} = 63.88^\circ$$ ans.

(iii) the unit vector in direction of $(\bar{A}+\bar{B}+\bar{C})$

solution

Given: $\bar{A} = 3\hat{i}+7\hat{j}-2\hat{k}$, $\bar{B} = \hat{i}+3\hat{j}+7\hat{k}$, $\bar{C} = 9\hat{i}-4\hat{j}+6\hat{k}$

$$(\bar{A}+\bar{B}+\bar{C}) = (3\hat{i}+7\hat{j}-2\hat{k}) + (\hat{i}+3\hat{j}+7\hat{k}) + (9\hat{i}-4\hat{j}+6\hat{k})$$

$$= 13\hat{i}+6\hat{j}+11\hat{k}$$

$$|A+B+C| = \sqrt{13^2+6^2+11^2} = \sqrt{169+36+121}$$

$$= \sqrt{326} = 18.06$$

$$e_{\bar{A}+\bar{B}+\bar{C}} = \frac{13\hat{i}+6\hat{j}+11\hat{k}}{18.06} = \frac{13}{18.06}\hat{i} + \frac{6}{18.06}\hat{j} + \frac{11}{18.06}\hat{k}$$

(2) A particle moves along a curve, $x = 8t^2$, $y = t^2 - 4t$, $z = t + 1$. where t is time. find the modulus of acceleration at $t = 1$

solution

when $x = 8t^2$, $y = t^2 - 4t$, $z = t + 1$

let $r = 8t^2 i + (t^2 - 4t) j + (t + 1) k$

since t = time

velocity $= \frac{dr}{dt} = 16t i + (2t - 4) j + k$

Acceleration $= \frac{d^2 r}{dt^2} = 16 i + 2 j$

$\left| \frac{d^2 r}{dt^2} \right|_{t=1} = \sqrt{16^2 + 2^2}$

$\therefore$ The modulus of its acceleration at $t$

$= 1 = \sqrt{256 + 4} = \sqrt{260} = 2\sqrt{65}$

3) If $A = 4i + 2j - 4k$, $B = 8i - 2j + k$, $C = i + 4j - 3k$, find the vector triple product $(A \times B) \times C$.

Solution

$A = 4i + 2j - 4k$, $B = 8i - 2j + k$, $C = i + 4j - 3k$

$$(A \times B) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$(A \times B) = i \begin{vmatrix} 2 & -4 \\ -2 & 1 \end{vmatrix} - j \begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k \begin{vmatrix} 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$= i(2-8) - j(4+32) + k(-8-16)$$

$$= i(-6) - j(36) + k(-24)$$

$$A \times B = -6i - 36j - 24k$$

2. $$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$(A \times B) \times C = i \begin{vmatrix} -36 & -24 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} -6 & -24 \\ 1 & -3 \end{vmatrix} + k \begin{vmatrix} -6 & -36 \\ 1 & 4 \end{vmatrix}$$

$$= i(108+96) - j(18+24) + k(-24+36)$$

$$= 204i - 42j + 12k$$

ans.