NAME: INNOCENT Obanele Iochukwu Date: 30/4/20
Matric no: 19/Eng 09/014 GENERAL MATHEMATICS III
Dept: AERONAUTICAL ENGINEERING MAT 106.
Assignment for Dr Okelamis Group

1) find the limit of the function $((4x^2 - \sin x)/x^3)$ as $x \to 0$.

Given $\frac{4x^2 - \sin x}{x^3}$, by direct substitution as $x$ tend to 0, we have:

$$\frac{4(0) - \sin 0}{0^3}$$

$$= \frac{0-0}{0} = \frac{0}{0} = \text{undefined}$$

use L'Hopital's rule:

$$\lim_{x \to 0} \left(\frac{4x^2 - \sin x}{x^3}\right) = \frac{8x - \cos x}{3x^2} = \frac{8x - \cos x}{3x^2}$$

i.e by differentiating:

$$\therefore \lim_{x \to 0} \left(\frac{8(0 - \cos 0}{3(0)^2}\right) = \frac{0-1}{0} = \frac{-1}{0} = \text{undefined}$$

$\therefore$ by futher differentiation:

$$\frac{8x - \cos x}{3x^2} = \frac{8 - (-\sin x)}{6x} = \frac{8 + \sin x}{6x}$$

$$\therefore \lim_{x \to 0} \left(\frac{8 + \sin x}{6x}\right) = \frac{8 + \sin 0}{6(0)} = \text{undefined}$$

By futher differentiation,

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$$\frac{8+\sin x}{6x} = \frac{\cos x}{6.}$$

$$\lim_{x \to 0} = \left(\frac{\cos 0}{6}\right) = \frac{1}{6}$$ //

2) If $y = (7x^2 \cos 8x)/e^{3x}$, find the derivative of $y$ with respect to $x$.

Solution

Given $y = \dfrac{7x^2 \cos 8x}{e^{3x}}$.

let $u = 7x^2$, $\dfrac{du}{dx} = 14x$,

$v = \cos 8x$, $\dfrac{dv}{dx} = -8\sin 8x$.

$w = e^{3x}$. $\dfrac{dw}{dx} = 3e^{3x}$.

$$\frac{dy}{dx} = y\left(\frac{1}{u} \times \frac{du}{dx} + \frac{1}{v}\times\frac{dv}{dx} - \frac{1}{w} \times \frac{dw}{dx}\right)$$

$$\therefore \frac{dy}{dx} = y\left(\frac{1}{7x^2_x} \times \overset{2}{14x} + \frac{1}{\cos 8x} \times -8\sin 8x - \frac{1}{e^{3x}} \times 3e^{3x}\right)$$

$$\frac{dy}{dx} = y\left(\frac{2}{x}' - \frac{8\sin 8x}{\cos 8x} - 3\right)$$

$$dy/dx = y\left(2/x - 8\tan 8x - 3\right)$$ //

3) If $y = \cos(5x^2 + 6x)$, find $dy/dx$ DATE

Solution.

Given $y = \cos(5x^2 + 6x)$.

let $u = 5x^2 + 6x$.

$\therefore \frac{du}{dx} = 10x + 6$

$\therefore y = \cos u$.

$\frac{dy}{du} = -\sin u$.

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = 10x + 6 \times -\sin u$.

$\frac{dy}{dx} = -\sin u \times 10x + 6$.

$\frac{dy}{dx} = -\sin u(10x + 6)$, but $u = 5x^2 + 6x$

$\therefore \frac{dy}{dx} = -\sin 5x^2 + 6x\ (10x + 6)$. //

4). Find the Integral of the following:

a) $\frac{3}{(4x+1)} dx$.

Solution.

$\int \frac{3}{4x+1} dx$, let $u = 4x + 1$, $\frac{du}{dx} = \frac{4}{1}$.

$\therefore dx = \frac{du}{4}$. but $x = \frac{u-1}{4}$

$\int \frac{du}{4} \times 3 \times \frac{1}{4x+1} = \int \frac{3\,du}{4(4x+1)}$

Integrating,

$$\therefore = \frac{3}{4}\int \frac{du}{4x+1} = \frac{3}{4}\int \frac{du}{u} = \frac{3}{4_2} \times \frac{u'}{u^2} + c$$

by final simplification, $= \frac{3}{2u^2} + C$

$$= \frac{3}{2(4x+1)^2} + C$$

b) $\frac{dx}{(x^2+49)}$

Solution.

$$\int \frac{dx}{(x^2+49)} = \int \frac{dx}{x^2+7^2}.$$

$$x = 7\tan\theta.$$

$$\frac{dx}{d\theta} = 7\sec^2\theta, \quad dx = 7\sec^2\theta\, d\theta$$

$$x^2+7^2 = 7\tan^2\theta + 7^2 = 7^2(\tan^2\theta+1).$$

factorizing after substituting.

$$\int \frac{7\sec^2\theta\, d\theta}{49\sec^2\theta} = \int \frac{d\theta}{7} = \frac{1}{7}\int d\theta = \frac{1}{7}\theta + c.$$

but $\theta = \tan^{-1}\frac{x}{7}.$

$$= \frac{1}{7}\tan^{-1}\frac{x}{7} + c$$

c) $(e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx.$

Solution:

$$\int e^{6x} + 9x - \sin 7x + \cos 8x \, dx$$

$$= \frac{1}{6}e^{6x} + \frac{9x^2}{2} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$$

d) $x\sqrt{9+x^2}\,dx$

Solution.

$$\int x\sqrt{9+x^2}\,dx.$$

let $u = 9+x^2$, $x = \sqrt{u-9}$

$\frac{du}{dx} = 2x$ $\therefore$ $dx = \frac{du}{2x}$

from $\int x\sqrt{9+x^2}\,dx$, substituting,

$$\int x\,u^{1/2}\,\frac{du}{2x}$$

$$\int\left(\sqrt{u-9}\right)u^{1/2}\,\frac{du}{2x} = \int (u-9)^{1/2}\,u^{1/2}\,\frac{du}{2\left((u-9)^{1/2}\right)}$$

$$= \frac{1}{2}\int u^{1/2}\,du.$$

Integrating

$$= \frac{1}{2}\left(\frac{u^{1/2+1}}{1/2+1}\right) = \frac{1}{2}\,\frac{u^{\frac{3}{2}}}{3/2}$$

$$= \frac{1}{2}\left(\frac{u^{3/2}}{1}\times\frac{2}{3}\right) = \frac{1}{\cancel{2}}\times\frac{\cancel{2}u^{3/2}}{3} = \frac{u^{3/2}}{3}. = \frac{1}{3}u^{3/2}\ //.$$