



AFE BABALOLA UNIVERSITY ADO-EKITI, EKITI STATE

Engineering Mathematics II
(ENG 282)

ASSIGNMENT V

SUBMITTED BY:
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MECHATRONICS ENGINEERING
18/ENG05/056

QUESTION 1

Solution:

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 18/ENG05/056
 Mechatronics Engineering
 ENG 282 Assignment V

1.)

From Balance Law,
 Accumulation Rate of salt into tank = Input rate of salt into tank - Output rate salt out of tank

$$\frac{dm}{dt} = m_{in} - m_{out} \quad \text{--- ①}$$

$$m_{in} = 50 \text{ gal/min} \times (1 + \sin t) \text{ lb/gal} = 50 (1 + \sin t) \text{ lb/min} \quad \text{--- ②}$$

$$m_{out} = \frac{30 \text{ gal}}{1200 \text{ gal}} \times 100 = 2.5\% \text{ of } m \quad \text{--- ③}$$

substitute for ② & ③ in ①

$$\frac{dm}{dt} = 50(1 + \sin t) - 2.5 \times m$$

$$= 50(1 + \sin t) - 0.025m$$

$$\frac{dm}{dt} = 50(1 + \sin t) - \frac{m}{40}$$

② $\frac{dm}{dt} + \frac{m}{40} = 50(1 + \sin t) \quad \text{--- ④}$

(b)

Solve using Integrating Factor, (IF)

$$\frac{dy}{dx} + Py = Q \quad \text{--- (5)}$$

comparing coefficients in (4) & (5)

$$P = \frac{1}{40}, \quad Q = 50(1 + \sin t)$$

$$IF = e^{\int P dx} = \int P dx = \int \frac{1}{40} dt = \frac{t}{40}$$

$$\therefore IF = e^{t/40}$$

$$m \cdot IF = \int Q \cdot IF dt$$

$$me^{t/40} = \int 50e^{t/40}(1 + \sin t) dt \quad \text{--- (6)}$$

Solving RHS,

$$50 \int e^{t/40}(1 + \sin t) dt \quad \text{--- (7)}$$

$$\text{let } t/40 = u; \quad t = 40u$$

$$\frac{dy}{dt} = \frac{1}{40}$$

$$dt = 40 du$$

substitute for dt in (7)

$$2000 \int e^u(1 + \sin 40u) du$$

Integrate by parts,

$$\int U dv = UV - \int V du$$

$$\text{let } u = \sin 40u, \quad dv = e^u$$

$$du = 40 \cos 40u, \quad v = e^u$$

$$\Rightarrow 2000 [e^u(\sin 40u + 1) - \int 40e^u \cos 40u du] \quad \text{--- (8)}$$

Solving for $\int 40e^u \cos 40u du$

Integrating by part,

$$\int U dv = UV - \int V du$$

u	dv	
cos 40u	e ^u	Integrate
-40 sin 40u	e ^u	
diff. = -160 cos 40u	e ^u	
Integrate	e ^u	

$$\Rightarrow e^u \cos 40u - (-40e^u \sin 40u) - \int -1600e^u \cos 40u du$$

$$\Rightarrow e^u \cos 40u - (-40e^u \sin 40u + 1600 \int e^u \cos 40u du)$$

Solve for $\int e^u \cos 40u du$ again, (10)

$$\Rightarrow 40e^u \sin 40u + e^u \cos 40u = 1600 \int e^u \cos 40u du \quad \text{--- (11)}$$

recall,

$$\int e^u \cos 40u du = \text{eqn (11)}$$

$$\therefore \Rightarrow 40 \int 40e^u \sin 40u + e^u \cos 40u = 1600 \int e^u \cos 40u du$$

substitute back in (8)

$$\Rightarrow 2000 [e^u(\sin 40u + 1) - 40 \frac{40e^u \sin 40u + e^u \cos 40u}{1600}]$$

$$= 2000e^u(\sin 40u + 1) - 8000 \frac{40e^u \sin 40u + e^u \cos 40u}{1600}$$

recall, $u = \frac{t}{40}$

$$= 2000e^{t/40}(\sin t + 1) - 8000 \frac{40e^{t/40} \sin t + e^{t/40} \cos t}{1600}$$

$$= 2000e^{t/40}(\sin t + 1) - 8000e^{t/40} \frac{(40 \sin t + \cos t)}{1600}$$

$$= 2000e^{t/40}[(\sin t + 1) - 40(40 \sin t + \cos t)]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

$$= 2000e^{t/40}[(\sin t + 1) - 1600 \sin t - 40 \cos t]$$

substitute back in (6)

$$m e^{t/40} = \frac{2000 e^{t/40} (\sin t - 40 \cos t + 1601)}{1601} + C$$

$e^{t/40}$ cancels out

$$m(t) = \frac{2000}{1601} (\sin t - 40 \cos t + 1601) + \frac{C}{e^{t/40}}$$

m is a function of t , and recall that there is 150 lb of salt in the tank at time, $t = 0$ minutes, \therefore

$$m(0) = \frac{2000}{1601} (\sin(0) - 40 \cos(0) + 1601) + C e^{-0/40}$$

$$150 = \frac{2000(-40 + 1601)}{1601} + C$$

$$150 = \frac{2000 \times 1561}{1601} + C$$

$$C \approx 150 - 1950.03$$

$$C \approx -1800.03$$

\therefore The amount of salt (m) at any given time (t) in the tank can be expressed as

$$\Rightarrow m(t) = \left(\frac{2000 (\sin t - 40 \cos t + 1601)}{1601} - \frac{1800.03}{e^{t/40}} \right) \text{ lb}$$

**When manual solution;

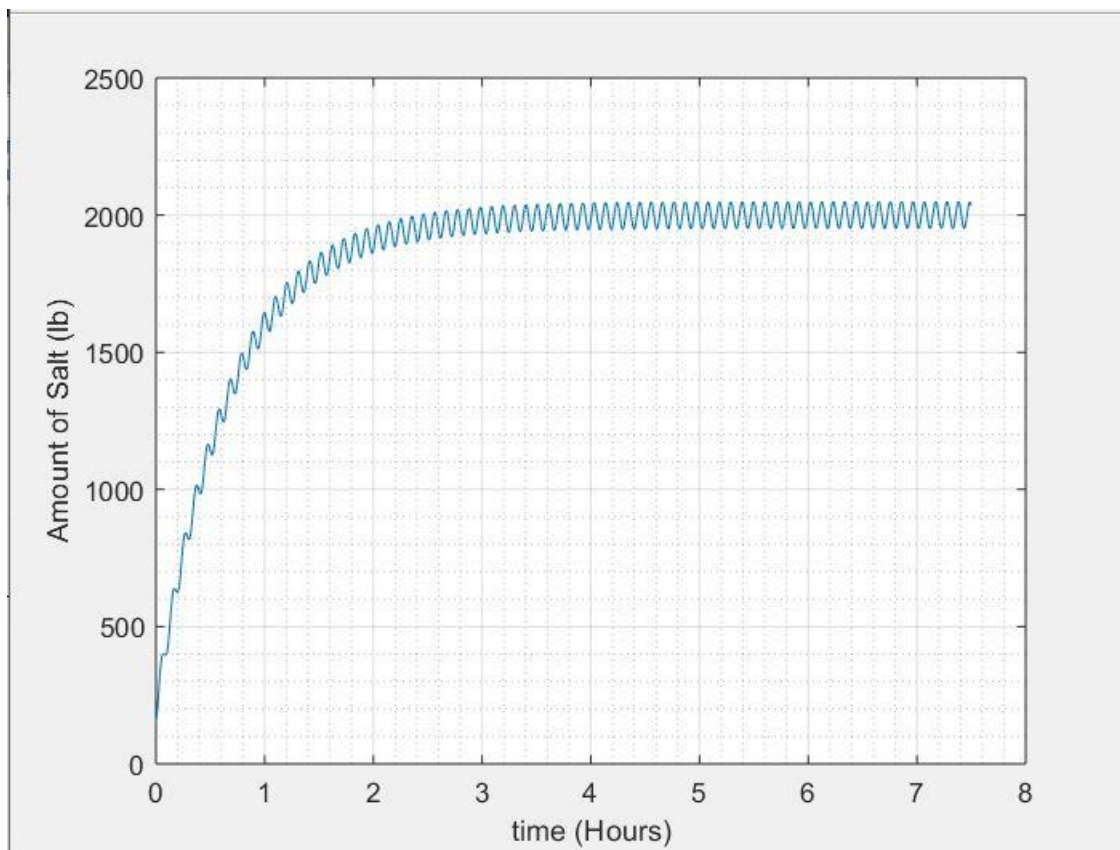
$$m(t) = (2000/1601) * (\sin(t) - 40 * \cos(t) + 1601) - 1800.03 * \exp(-t/40)$$

is plotted against time, it yields the same graph as the code

Code:

```
AssignmentV_1_Modelling_using_dsolve.m x AssignmentV_2_Graph_and_xlswrite_ForLoop_and_IfCondition.m x +
1 -  commandwindow
2 -  clear
3 -  clc
4 -  close all
5
6 -  syms m(t)
7
8 -  m_in = 50*(1 + sin(t)) %rate at which salt enters tank (lb/min)
9 -  m_out = (30/1200)*m %rate at which salt leaves the tank (lb/min)
10 - accumulation_rate = diff(m, t) %differential equation (net rate of salt accumulation in tank)
11 - m = dsolve(accumulation_rate == m_in - m_out, m(0) == 150) %amount of salt in tank at time=0 is 150lb
12
13 % 0 < t <= 7.5 hours
14 % 0.5 minutes = (0.5 / 60) hours
15 - t = 0:(0.5/60):7.5
16 - m_num = subs(m, t*60)
17
18 - plot(t, m_num)
19 - grid on
20 - grid minor
21 - xlabel('time (Hours)')
22 - ylabel('Amount of Salt (lb)')
```

Graph:



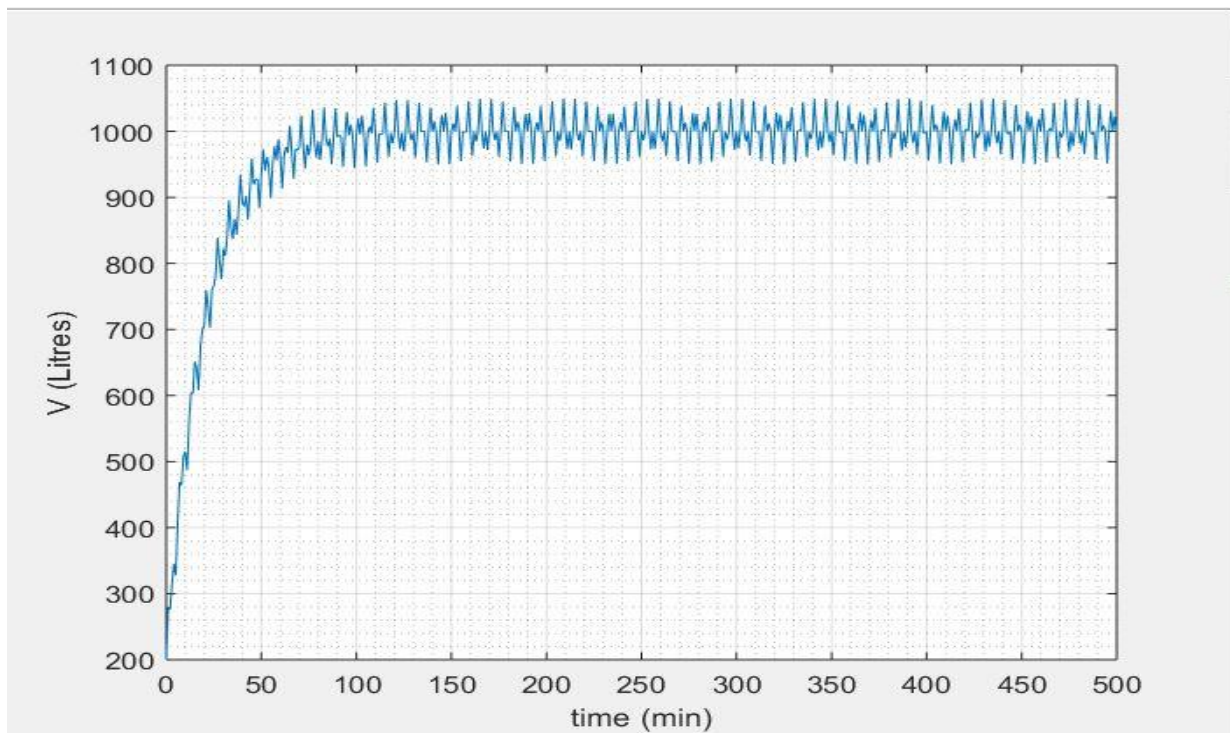
Graph of Amount of Salt (lb) against time (hours)

QUESTION 2

Code:

```
AssignmentV_1_Modelling_using_dsolve.m  AssignmentV_2_Graph_and_xlsxwrite_ForLoop_and_IfCondition.m  +
1-  commandwindow
2-  clear
3-  clc
4-  close all
5-
6-  time = []
7-  responses = []
8-
9-  %this loop calculates the the responses for y and y_mean at odd and even
10 %number time values
11 for t = 0:1:500
12     if mod(t,2) == 0
13         time = [time: t] %time is an array of time values from 0 to 500 minutes with a step size of 1 minnute
14         y_mean = 1000 - (800*exp(-0.05*t))
15         responses = [responses; y_mean] %responses is an array of combines y and y_mean responses corresponding to time values
16     else
17         time = [time: t]
18         y = (50/0.05) + (50/1.0025)*sin(t) + ((50 * 0.05)/1.0025)*cos(t) - 802.49*exp(-0.05*t)
19         responses = [responses; y]
20     end
21 end
22
23
24 plot(time, responses)
25 grid on
26 grid minor
27 xlabel('time (min)')
28 ylabel('V (Litres)')
29
30
31 xlsxwrite('odevbesdata.xlsx', {'t (min)'}, 'Veriler', 'A1')
32 xlsxwrite('odevbesdata.xlsx', {'V (Litres)'}, 'Veriler', 'B1')
33 xlsxwrite('odevbesdata.xlsx', time, 'Veriler', 'A2')
34 xlsxwrite('odevbesdata.xlsx', responses, 'Veriler', 'B2')
```

Graph:



Graph of V (Litres) against time (min)

Spreadsheet:

oDevbesdata.xlsx - Excel

File Home Insert Page Layout Formulas Data Review View Help Search

Clipboard Font Alignment Number Styles Cells Editing

Normal Bad Good Neutral Calculation Check Cell

AutoSum Fill Sort & Filter Find & Select

UPDATE AVAILABLE Updates for Office are ready to be installed, but first we need to close some apps. Update now

Sheet1 Veriler

Average: 608.6595942 Count: 1004 Sum: 609876.9134

Ready

Name of workbook, "odevbesdata"

Name of spreadsheet, "Veriler"

Table Data:

t (min)	V (Litres)
0	200
1	279.9639
2	276.1301
3	313.8601
4	345.0154
5	327.9009
6	407.3454
7	469.1423
8	463.744
9	506.5922
10	514.7755
11	487.1398
12	560.9507
13	604.2824
14	602.7318
15	651.4694
16	640.5368
17	608.3676
18	674.7443
19	699.585
20	705.6964
21	759.541
22	733.7031

23	702.3679
24	759.0446
25	765.9535
26	781.9746
27	838.9333
28	802.7224
29	776.7953
30	821.4959
31	811.8028
32	838.4828
33	895.7197
34	853.8532
35	836.9388
36	867.7609
37	843.6308
38	880.3451
39	934.561
40	891.7318
41	886.318
42	902.0349
43	866.4219
44	911.3575
45	959.1673
46	919.7929
47	927.1558
48	927.4256
49	883.9312
50	934.332
51	972.6189
52	940.5811
53	960.76
54	946.2356
55	898.8906
56	951.3519
57	977.5784
58	955.9814
59	987.8325
60	960.1703
61	913.1659
62	963.9606
63	976.4171
64	967.3902
65	1008.72
66	970.4935
67	927.8846
68	973.3014

69	971.2768
70	975.8421
71	1023.612
72	978.141
73	943.5523
74	980.2212
75	964.0851
76	982.1034
77	1032.697
78	983.8065
79	960.1637
80	985.3475
81	956.5397
82	986.7419
83	1036.269
84	988.0035
85	977.3165
86	989.1452
87	950.0748
88	990.1781
89	1034.797
90	991.1128
91	994.3264
92	991.9585
93	945.8226
94	992.7238
95	1028.956
96	993.4162
97	1010.344
98	994.0427
99	944.5792
100	994.6096
101	1019.626
102	995.1226
103	1024.467
104	995.5867
105	946.7823
106	996.0067
107	1007.857
108	996.3867
109	1035.849
110	996.7306
111	952.5074
112	997.0417
113	994.8122
114	997.3232

115	1043.787
116	997.578
117	961.4843
118	997.8084
119	981.7003
120	998.017
121	1047.803
122	998.2057
123	973.1357
124	998.3765
125	969.69
126	998.531
127	1047.688
128	998.6708
129	986.6355
130	998.7972
131	959.8303
132	998.9117
133	1043.536
134	999.0153
135	1000.984
136	999.109
137	952.9749
138	999.1938
139	1035.738
140	999.2705
141	1015.094
142	999.3399
143	949.7208
144	999.4027
145	1024.963
146	999.4596
147	1027.883
148	999.511
149	950.3645
150	999.5575
151	1012.102
152	999.5996
153	1038.363
154	999.6377
155	954.8828
156	999.6722
157	998.2056
158	999.7034
159	1045.722
160	999.7316

161	962.9361
162	999.7572
163	984.3985
164	999.7803
165	1049.39
166	999.8012
167	973.8983
168	999.8201
169	971.7947
170	999.8372
171	1049.089
172	999.8527
173	986.9075
174	999.8667
175	961.4085
176	999.8794
177	1044.851
178	999.8909
179	1000.936
180	999.9013
181	954.0748
182	999.9107
183	1037.02
184	999.9192
185	1014.872
186	999.9269
187	950.3834
188	999.9338
189	1026.226
190	999.9401
191	1027.61
192	999.9458
193	950.6326
194	999.951
195	1013.332
196	999.9556
197	1038.139
198	999.9599
199	954.8056
200	999.9637
201	999.3675
202	999.9671
203	1045.622
204	999.9703
205	962.5722
206	999.9731

207	985.448
208	999.9757
209	1049.466
210	999.978
211	973.3154
212	999.9801
213	972.6833
214	999.982
215	1049.366
216	999.9837
217	986.1808
218	999.9852
219	962.0913
220	999.9866
221	1045.33
222	999.9879
223	1000.144
224	999.9891
225	954.5168
226	999.9901
227	1037.681
228	999.991
229	1014.094
230	999.9919
231	950.5636
232	999.9927
233	1027.029
234	999.9934
235	1026.92
236	999.994
237	950.5472
238	999.9946
239	1014.223
240	999.9951
241	1037.601
242	999.9956
243	954.4692
244	999.996
245	1000.282
246	999.9964
247	1045.285
248	999.9967
249	962.0175
250	999.997
251	986.3189
252	999.9973

253	1049.361
254	999.9976
255	972.5908
256	999.9978
257	973.4447
258	999.998
259	1049.505
260	999.9982
261	985.3472
262	999.9984
263	962.6855
264	999.9985
265	1045.705
266	999.9987
267	999.2705
268	999.9988
269	954.8986
270	999.9989
271	1038.264
272	999.999
273	1013.252
274	999.9991
275	950.7042
276	999.9992
277	1027.774
278	999.9993
279	1026.177
280	999.9993
281	950.4365
282	999.9994
283	1015.072
284	999.9995
285	1037.017
286	999.9995
287	954.1169
288	999.9996
289	1001.17
290	999.9996
291	1044.908
292	999.9996
293	961.4523
294	999.9997
295	987.1737
296	999.9997
297	1049.222
298	999.9997

299	971.8582
300	999.9998
301	974.1994
302	999.9998
303	1049.615
304	999.9998
305	984.506
306	999.9998
307	963.2803
308	999.9998
309	1046.055
310	999.9999
311	998.3879
312	999.9999
313	955.2862
314	999.9999
315	1038.827
316	999.9999
317	1012.398
318	999.9999
319	950.854
320	999.9999
321	1028.505
322	999.9999
323	1025.421
324	999.9999
325	950.3367
326	999.9999
327	1015.913
328	999.9999
329	1036.418
330	999.9999
331	953.7756
332	1000
333	1002.054
334	1000
335	1044.515
336	1000
337	960.8966
338	1000
339	988.0303
340	1000
341	1049.066
342	1000
343	971.1326
344	1000

345	974.9605
346	1000
347	1049.708
348	1000
349	983.6682
350	1000
351	963.8853
352	1000
353	1046.39
354	1000
355	997.5047
356	1000
357	955.687
358	1000
359	1039.377
360	1000
361	1011.54
362	1000
363	951.0186
364	1000
365	1029.227
366	1000
367	1024.656
368	1000
369	950.252
370	1000
371	1016.749
372	1000
373	1035.808
374	1000
375	953.4484
376	1000
377	1002.937
378	1000
379	1044.107
380	1000
381	960.353
382	1000
383	988.8905
384	1000
385	1048.893
386	1000
387	970.4158
388	1000
389	975.7293
390	1000

391	1049.785
392	1000
393	982.8354
394	1000
395	964.5015
396	1000
397	1046.71
398	1000
399	996.6222
400	1000
401	956.1015
402	1000
403	1039.914
404	1000
405	1010.678
406	1000
407	951.1984
408	1000
409	1029.939
410	1000
411	1023.883
412	1000
413	950.1829
414	1000
415	1017.579
416	1000
417	1035.186
418	1000
419	953.1357
420	1000
421	1003.819
422	1000
423	1043.686
424	1000
425	959.8217
426	1000
427	989.754
428	1000
429	1048.706
430	1000
431	969.7083
432	1000
433	976.5057
434	1000
435	1049.846
436	1000

437	982.0079
438	1000
439	965.1288
440	1000
441	1047.015
442	1000
443	995.7408
444	1000
445	956.5298
446	1000
447	1040.439
448	1000
449	1009.813
450	1000
451	951.3936
452	1000
453	1030.642
454	1000
455	1023.103
456	1000
457	950.1293
458	1000
459	1018.404
460	1000
461	1034.553
462	1000
463	952.8377
464	1000
465	1004.699
466	1000
467	1043.251
468	1000
469	959.303
470	1000
471	990.6208
472	1000
473	1048.503
474	1000
475	969.0103
476	1000
477	977.2894
478	1000
479	1049.892
480	1000
481	981.1861
482	1000

483	965.767
484	1000
485	1047.306
486	1000
487	994.8607
488	1000
489	956.9717
490	1000
491	1040.952
492	1000
493	1008.945
494	1000
495	951.6039
496	1000
497	1031.335
498	1000
499	1022.316
500	1000