

NAME : ARIKEKPAR AZIBAOLA ISRAEL

DEPT : AERONAUTICAL & ASTRONAUTICAL ENGINEERING

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ASSIGNMENT FOR DR. GIELAMIS GROUP

Find the limit of the function $\frac{(4x^3 - \sin x)}{x^3}$ as $x \rightarrow 0$.

Solution.

Given $\frac{4x^3 - \sin x}{x^3}$, by direct substitution as x tends to 0, we have:

$$\frac{4(0) - \sin 0}{0^3} = \frac{0 - 0}{0} = \frac{0}{0} = \text{undefined}$$

Use L'Hopital's rule.

$$\lim_{x \rightarrow 0} \left(\frac{4x^3 - \sin x}{x^3} \right) = \frac{8x^2 - (\cos x)}{3x^2} = \frac{8x - (\cos x)}{3x^2}$$

if by differentiating:

$$\therefore \lim_{x \rightarrow 0} \left(\frac{8(0) - (\cos 0)}{3(0)^2} \right) = \frac{0 - 1}{0} = \frac{-1}{0} = \text{undefined}$$

\therefore by further differentiation:

$$\frac{8x - (\cos x)}{3x^2} = \frac{8 - (-\sin x)}{6x} = \frac{8 + \sin x}{6x}$$
$$\therefore \lim_{x \rightarrow 0} \left(\frac{8 + \sin x}{6x} \right) = \frac{8 + \sin 0}{6(0)} = \text{undefined}$$

By further differentiation,

DATE

$$\frac{8 + \sin x}{6x} = \frac{\cos x}{6}$$

$$\lim_{x \rightarrow 0} = \left(\frac{\cos 0}{6} \right) = \frac{1}{6}$$

2) If $y = (7x^2 \cos 8x) / e^{3x}$, find the derivative of y with respect to x .

Solution

$$\text{Given } y = \frac{7x^2 \cos 8x}{e^{3x}}$$

$$\text{let } u = 7x^2, \quad \frac{du}{dx} = 14x$$

$$v = \cos 8x, \quad \frac{dv}{dx} = -8 \sin 8x$$

$$w = e^{3x}, \quad \frac{dw}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = \frac{1}{w} \left(u \frac{dv}{dx} + v \frac{du}{dx} - \frac{1}{w} \times \frac{dw}{dx} \right)$$

$$\frac{dy}{dx} = \frac{1}{e^{3x}} \left(\frac{1}{7x^2} \times 14x + \frac{1}{\cos 8x} \times -8 \sin 8x - \frac{1}{e^{3x}} \times 3e^{3x} \right)$$

$$\frac{dy}{dx} = \frac{1}{e^{3x}} \left(\frac{2}{x} - \frac{8 \sin 8x}{\cos 8x} - 3 \right)$$

$$\frac{dy}{dx} = \frac{1}{e^{3x}} (2/x - 8 \tan 8x - 3)$$

3) If $y = \cos(5x^2 + 6x)$, find dy/dx DATE _____
solution.

$$\text{Given } y = \cos(5x^2 + 6x).$$

$$\text{let } u = 5x^2 + 6x.$$

$$\therefore \frac{du}{dx} = 10x + 6$$

$$\therefore y = \cos u.$$

$$\frac{dy}{du} = -\sin u.$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 10x + 6 \times -\sin u.$$

$$\frac{dy}{dx} = -\sin u \times 10x + 6.$$

$$\frac{dy}{dx} = -\sin u (10x + 6), \text{ but } u = 5x^2 + 6x$$

$$\therefore \frac{dy}{dx} = -\sin(5x^2 + 6x) (10x + 6) //$$

4) Find the Integral of the following:

a) $\int \frac{3}{4x+1} dx$.

(4x+1)

solution.

$$\int \frac{3}{4x+1} dx, \text{ let } u = 4x+1, \frac{du}{dx} = 4.$$

$$\therefore dx = \frac{du}{4} \text{ but } x = \frac{u-1}{4}$$

$$\int \frac{du}{4} \times 3 \times \frac{1}{4x+1} = \int \frac{3du}{4(4x+1)}$$

Integrating,

$$= \frac{3}{4} \int \frac{du}{u+1} = \frac{3}{4} \int \frac{du}{u} = \frac{3}{4} \times \frac{2u}{u^2} + C$$

by final simplification, $= \frac{3}{2} + C$
 $= \frac{3}{2(4+x^2)} + C$

b) $\frac{dx}{(x^2+49)}$

Solution:

$$\int \frac{dx}{(x^2+49)} = \int \frac{dx}{x^2+7^2}$$

$$x = 7 \tan \theta$$

$$\frac{dx}{d\theta} = 7 \sec^2 \theta, \quad dx = 7 \sec^2 \theta d\theta$$

$$x^2 + 7^2 = 7 \tan^2 \theta + 7^2 = 7^2 (\tan^2 \theta + 1)$$

Integrating after substituting.

$$\int \frac{7 \sec^2 \theta d\theta}{7^2 \sec^2 \theta} = \int \frac{d\theta}{7} = \frac{1}{7} \int \frac{d\theta}{1} + C$$

$$\text{but } \theta = \tan^{-1} \frac{x}{7}$$

$$= \frac{1}{7} \tan^{-1} \frac{x}{7} + C$$

c) $(e^{6x} + 9x^2 - \sin 7x + \cos 8x) dx$

Solution:

$$\int (e^{6x} + 9x^2 - \sin 7x + \cos 8x) dx$$

$$= \frac{1}{6} e^{6x} + \frac{9x^3}{3} + \frac{1}{7} \cos 7x + \frac{1}{8} \sin 8x + C$$

$$d) \int x\sqrt{9+x^2} dx$$

Solution
$$\int x\sqrt{9+x^2} dx$$

$$\text{let } u = 9+x^2, \quad x = \sqrt{u-9}$$

$$\frac{du}{dx} = 2x \quad \therefore dx = \frac{du}{2x}$$

from $\int x\sqrt{9+x^2} dx$, substituting,

$$\int x u^{1/2} \frac{du}{2x}$$

$$\int (\sqrt{u-9}) u^{1/2} \frac{du}{2x} = \int (\cancel{\sqrt{u-9}})^{1/2} u^{1/2} \frac{du}{2(\cancel{u-9})^{1/2}}$$

$$= \frac{1}{2} \int u^{1/2} du$$

Integrating
$$= \frac{1}{2} \left(\frac{u^{1/2+1}}{1/2+1} \right) = \frac{1}{2} \frac{u^{3/2}}{3/2}$$

$$= \frac{1}{2} \left(\frac{u^{3/2}}{1} \times \frac{2}{3} \right) = \frac{1}{2} \times \frac{2}{3} u^{3/2} = \frac{u^{3/2}}{3} = \frac{1}{3} u^{3/2}$$