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COVID-19 HOLIDAY ASSIGNMENT

QUESTION 1

Find the limit of the function $\frac{4x^2 - \sin(x)}{x^3}$ as $x \rightarrow 0$

SOLUTION

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{1}{x^3} (4x^2 - \sin(x)) \\ &= \lim_{x \rightarrow 0} \frac{1}{x^3} (4x^2 - \sin(x)) \end{aligned}$$

Since we have an indeterminate form of type $\frac{0}{0}$, then we can apply the l'Hopital's rule:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{1}{x^3} (4x^2 - \sin(x)) \\ &= \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(4x^2 - \sin(x))}{\frac{d}{dx}(x^3)} \\ &= \lim_{x \rightarrow 0} \frac{1}{3x^2} (8x - \cos(x)) \end{aligned}$$

Apply the constant multiple rule

$$\begin{aligned} & \lim_{x \rightarrow 0} cf(x) = c \lim_{x \rightarrow 0} f(x) \text{ with } c = \frac{1}{3} \text{ and } f(x) = \frac{1}{x^2} (8x - \cos(x)): \\ & \lim_{x \rightarrow 0} \frac{1}{3x^2} (8x - \cos(x)) \\ &= \left(\frac{1}{3}\right) \lim_{x \rightarrow 0} \frac{1}{x^2} (8x - \cos(x)) \end{aligned}$$

The function decreases without a bound:

$$\lim_{x \rightarrow 0} \frac{1}{x^2} (8x - \cos(x)) = -\infty$$

Therefore, $\lim_{x \rightarrow 0} \frac{1}{x^3} (4x^2 - \sin(x)) = -\infty$

QUESTION 2

If $y = \frac{(7x^2 \cos(8x))}{e^{3x}}$. Find the derivative of y with respect to x .

SOLUTION

$$\begin{aligned} y &= (7x^2 e^{-3x} \cos(8x)) \\ \frac{d}{dx} [7x^2 e^{-3x} \cos(8x)] &= 7 \cdot \frac{d}{dx} [x^2 e^{-3x} \cos(8x)] \\ &= 7 \left(\frac{d}{dx} [x^2] \cdot e^{-3x} \cos(8x) + x^2 \cdot \frac{d}{dx} [e^{-3x}] \cdot \cos(8x) + x^2 e^{-3x} \cdot \frac{d}{dx} [\cos(8x)] \right) \\ &= 7(2x e^{-3x} \cos(8x) + x^2 e^{-3x} \cdot \frac{d}{dx} [-3x] \cdot \cos(8x) + x^2 e^{-3x} (-\sin(8x)) \cdot \frac{d}{dx} [8x]) \\ &= 7(2x e^{-3x} \cos(8x) + x^2 e^{-3x} (-3 \cdot \frac{d}{dx} [x]) \cos(8x) - x^2 e^{-3x} \cdot 8 \cdot \frac{d}{dx} [x] \cdot \sin(8x)) \\ &= 7(2x e^{-3x} \cos(8x) - 3x^2 e^{-3x} \cdot 1 \cos(8x) - 8x^2 e^{-3x} \cdot 1 \sin(8x)) \end{aligned}$$

$$\begin{aligned}
&= 7(-8x^2 e^{-3x} \sin(8x) - 3x^2 e^{-3x} \cos(8x) + 2x e^{-3x} \cos(8x)) \\
&\therefore \frac{d}{dx} [7x^2 e^{-3x} \cos(8x)] = -7x e^{-3x} (8x \sin(8x) + (3x - 2) \cos(8x))
\end{aligned}$$

QUESTION 3

If $y = \cos(5x^2 + 6x)$, find $\frac{dy}{dx}$.

SOLUTION

$$\begin{aligned}
y &= \cos(5x^2 + 6x) \\
&= \frac{d}{dx} [\cos(5x^2 + 6x)] \\
&= (-\sin(5x^2 + 6x)) \cdot \frac{d}{dx} [5x^2 + 6x] \\
&= -\left(5 \cdot \frac{d}{dx}[x^2] + 6 \cdot \frac{d}{dx}[x]\right) \sin(5x^2 + 6x) \\
&= -(5 \cdot 2x + 6 \cdot 1) \sin(5x^2 + 6x) \\
&= -(10x + 6) \sin(5x^2 + 6x) \\
&\therefore \frac{d}{dx} [\cos(5x^2 + 6x)] = -(10x + 6) \sin(5x^2 + 6x)
\end{aligned}$$

QUESTION 4

Find the integral of the following:

- (a) $\int \frac{3dx}{(4x+1)}$
- (b) $\int \frac{dx}{(x^2+49)}$
- (c) $\int (e^{6x} + 9x^3 - \sin(7x) + \cos(8x)) dx$
- (d) $\int x \sqrt{(9+x^2)} dx$

SOLUTION

$$\begin{aligned}
(a) \quad &\int \frac{3dx}{(4x+1)} \\
&= \frac{3}{(4x+1)} dx \\
&= \int \frac{3}{(4x+1)} dx
\end{aligned}$$

Substitute $u = 4x + 1$

$$\text{But } \frac{du}{dx} = 4$$

$$\text{And } dx = \frac{1}{4} du$$

$$\text{This implies that } \int \frac{3}{(4x+1)} dx = \int \frac{3}{u} \cdot \frac{1}{4} du$$

$$\int \frac{3}{u} \cdot \frac{1}{4} du = \frac{3}{4} \int \frac{1}{u} du$$

$$\text{But } \int \frac{1}{u} du = \ln(u)$$

$$\begin{aligned}
\text{Hence } \frac{3}{4} \int \frac{1}{u} du &= \frac{3}{4} \ln(u) + C \\
&= \frac{3}{4} \ln(4x+1) + C
\end{aligned}$$

$$\therefore \int \frac{3}{(4x+1)} dx = \frac{3}{4} \ln(4x+1) + C$$

$$\begin{aligned}
(b) \quad & \frac{dx}{(x^2 + 49)} \\
& \frac{1}{(x^2 + 49)} dx \\
& \int \frac{1}{(x^2 + 49)} dx \\
& \text{Substitute } u = \frac{x}{7}, x = 7u \\
& \text{But } \frac{du}{dx} = \frac{1}{7} \\
& \text{And } dx = 7du \\
& = \int \frac{7}{49x^2 + 49} du \\
& = \frac{1}{7} \int \frac{1}{u^2 + 1} du \\
& \text{But } \int \frac{1}{u^2 + 1} du = \tan^{-1}(u) \\
& \text{Hence } \frac{1}{7} \int \frac{1}{u^2 + 1} du = \frac{1}{7} \tan^{-1}(u) + C \\
& \qquad \qquad \qquad = \frac{1}{7} \tan^{-1}\left(\frac{x}{7}\right) + C \\
& \therefore \int \frac{1}{(x^2 + 49)} dx = \frac{1}{7} \tan^{-1}\left(\frac{x}{7}\right) + C
\end{aligned}$$

$$\begin{aligned}
(c) \quad & (e^{6x} + 9x^3 - \sin(7x) + \cos(8x)) dx \\
& = \int (e^{6x} + 9x^3 - \sin(7x) + \cos(8x)) dx \\
& = \int e^{6x} dx + \int 9x^3 dx - \int \sin(7x) dx + \int \cos(8x) dx \\
& = \int e^{6x} dx + 9 \int x^3 dx - \int \sin(7x) dx + \int \cos(8x) dx
\end{aligned}$$

But solving $\int e^{6x} dx$

Substitute $u = 6x$

$$\begin{aligned}
& \frac{du}{dx} = 6 \\
& dx = \frac{1}{6} du \\
& \int e^{6x} dx = \frac{1}{6} \int e^u du \\
& \text{But } \int e^u du = \frac{e^u}{\ln(e)} = e^u \\
& \text{This implies that } \frac{1}{6} \int e^u du = \frac{e^u}{6} = \frac{e^{6x}}{6}
\end{aligned}$$

But solving $9 \int x^3 dx$

$$\begin{aligned}
& \int x^3 dx \\
& = \frac{x^4}{4}
\end{aligned}$$

$$\text{This implies that } 9 \int x^3 dx = \frac{9x^4}{4}$$

But solving $\int \sin(7x) dx$

Substitute $u = 7x$

$$\begin{aligned}
& \frac{du}{dx} = 7 \\
& dx = \frac{1}{7} du
\end{aligned}$$

$$\int \sin(7x) dx = \frac{1}{7} \int \sin(u) du$$

But $\int \sin(u) du = -\cos(u)$

This implies that $\frac{1}{7} \int \sin(u) du = -\frac{\cos(u)}{7} = -\frac{\cos(7x)}{7}$

But solving $\int \cos(8x) dx$

Substitute $u = 8x$

$$\frac{du}{dx} = 8$$

$$dx = \frac{1}{8} du$$

$$\int \cos(8x) dx = \frac{1}{8} \int \cos(u) du$$

But $\int \cos(u) du = \sin(u)$

This implies that $\frac{1}{8} \int \cos(u) du = \frac{\sin(u)}{8} = \frac{\sin(8x)}{8}$

$$\text{Thus } \int e^{6x} dx + 9 \int x^3 dx - \int \sin(7x) dx + \int \cos(8x) dx = \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{\cos(7x)}{7} + \frac{\sin(8x)}{8}$$

$$\int (e^{6x} + 9x^3 - \sin(7x) + \cos(8x)) dx = \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{\cos(7x)}{7} + \frac{\sin(8x)}{8} + C$$

$$\therefore \int (e^{6x} + 9x^3 - \sin(7x) + \cos(8x)) dx = \frac{28e^{6x} + 378x^4 + 24\cos(7x) + 21\sin(8x)}{168} + C$$

$$(d) x\sqrt{(9+x^2)} dx$$

$$= \int x\sqrt{(9+x^2)} dx$$

Substitute $u = x^2 + 9$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{1}{2x} du$$

$$= \frac{1}{2} \int \sqrt{u} du$$

$$\text{But solving } \int \sqrt{u} du = \int u^{\frac{1}{2}} du$$

$$= \frac{2u^{\frac{3}{2}}}{3}$$

$$\frac{1}{2} \int \sqrt{u} du = \frac{1}{2} \cdot \frac{2u^{\frac{3}{2}}}{3} = \frac{u^{\frac{3}{2}}}{3} = \frac{(x^2+9)^{\frac{3}{2}}}{3}$$

$$\therefore \int x\sqrt{(9+x^2)} dx = \frac{(x^2+9)^{\frac{3}{2}}}{3} + C$$