1 $\int \frac{11-3x}{x^2+2x-3}\,dx$

$x^2+2x-3 = (x-1)(x+3)$

$\int \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$\int \frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$

$11-3x = Ax+3A+Bx-B$

$11-3x = Ax+Bx+3A-B.$

$11-3x = x(A+B)+3A-B.$

$A+B = -3$

$+\ 3A-B = 11$

$\frac{4A}{4} = \frac{8}{4}$

$A = 2.$

$A+B = -3$

$2+B = -3$

$B = -5.$

$\frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} + \frac{-5}{x+3}.$

$\int \frac{11-3x}{(x-1)(x+3)}\,dx = \int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx$

$\int \frac{11-3x}{(x-1)(x+3)}\,dx = 2\ln\{x-1\} - 5\ln\{x+3\} + c$

2. $\int \frac{4x-16}{x^2-2x-3}$.

$x^2-2x-3=(x+1)(x-3)$

$\int \frac{4x-16}{x^2-2x-3} = \int \frac{4x-16}{(x+1)(x-3)}$

$\frac{4x-16}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$

$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3)+B(x+1)}{(x+1)(x-3)}$

$4x-16 = Ax-3A+Bx+B.$

$4x-16 = Ax+Bx-3A+B.$

$4x-16 = x(A+B)-3A+B.$

$A+B=4$

$-\quad -3A+B=-16$

$\frac{4A}{4} = \frac{20}{4}$

$A=5.$

$A+B=4$

$5+B=4$

$B=-1$

$\int \frac{4x-16}{(x+1)(x-3)}dx = \int \frac{5}{x+1}dx + \int \frac{-1}{x-3}dx$

$\int \frac{4x-16}{(x+1)(x-3)}dx = 5\ln(x+1) - \ln(x-3) + C$

3. $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx.$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{x+3}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x-2)(x+3}$$

$2x^2-9x-35 = A[x^2+4x-6]+B[x^2+4x+3]+C[x^2-x-2]$

$2x^2-9x-35 = Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C$

$2x^2-9x-35 = Ax^2+Bx^2+Cx^2+Ax+4Bx-Cx-6A+3B-2C$

$2x^2-9x-35 = x^2[A+B+C]+x[A+4B-C]-6A+3B-2C$

$A+B+C=2$ -- eq ①

$A+4B-C=-9$ -- eq ②

$-6A+3B-2C=-35$ -- eq ③

$A=2-B-C$ -- eq ④ , Put eq 4 into 2 & 3

$A+4B-C=-9$

$2-B-C+4B-C=-9$

$3B-2C=-11$ -- eq ⑤

$-6A+3B-2C=-35$

$-6[2-B-C]+3B-2C=-35$

$-12+6B+6C+3B-2C=-35.$

$9B+4C=-23$ -- eq ⑥

eq ⑤ & ⑥

$3B-2C=-11 \times 4$

$9B+4C=-23. \times 2.$

$12B-8C=-44$

$+\ 18B+8C=-46$

$\frac{30B}{30}=\frac{-90}{30}$

$B=-3$

$3B - 2C = -11$

$3(-3) - 2C = -11$

$-9 - 2C = -11$

$-2C = -11 + 9$

$\frac{-2C}{-2} = \frac{-2}{-2}$

$C = 1$

$A = 2 - B - C$

$A = 2 - (-3) - 1$

$A = 4$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{(x+1)} + \frac{-3}{(x-2)} + \frac{1}{(x+3)}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4}{(x+1)}\,dx + \int \frac{-3}{(x-2)} + \int \frac{1}{(x+3)}\,dx$$

$$\int \frac{2x^2 - 9x - 35\,dx}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$$