NAME: ATUME MIMIDOO VICTORIA

MATRICULATION NUMBER: 19/MHS01/107

DEPARTMENT: MEDICINE AND SURGERY

COLLEGE: MEDICINE AND HEALTH SCIENCES

COURSE: MAT 107

ASSIGNMENT.

1. Simplify $\int \frac{11-3x}{x^2+2x-3} dx$

Solution

Factorizing the denominator $(x^2+2x-3) = (x-1)(x+3)$

$$\frac{11-3x}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{11-3x}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

Multiplying all by $(x-1)(x+3)$;

$$11-3x = A(x+3) + B(x-1)$$

~~A~~ To get the value of A

$$f(1) = 11-3(1) = A(1+3)$$

$$= 8 = 4A \quad ; \quad A = 2.$$

To get the value of B

$$f(-3) = 11-3(-3) = B(-3-1)$$

$$11+9 = -4B$$

$$B = 20/-4 = -5$$

Substituting the values of A and B:

$$\frac{11-3x}{x^2+2x-3} = \frac{2}{x-1} - \frac{5}{x+3}$$

$$\int \frac{11-3x}{x^2+2x-3} dx = \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$$

$\int \frac{2}{x-1} dx$; $u = x-1$, $du/dx = 1$, $dx = du$; $\int \frac{2}{x-1} dx = 2\int \frac{dx}{x-1}$

$$\int \frac{2}{x-1} dx = 2\int \frac{du}{u} = 2\ln u.$$

$\int \frac{5}{x+3} dx$; $u = x+3$ $\quad \int \frac{5}{x+3} dx = 5\int \frac{du}{u} = 5\ln u$

$du/dx = 1$

$du = dx$

$2\ln u - 5\ln u + c = 2\ln(x-1) - 5\ln(x+3) + c$

$\therefore \int \frac{11-3x}{x^2+2x-3} = 2\ln(x-1) - 5\ln(x+3) + c$ where c is the constant of integration.

② Simplify $\int \frac{4x-16}{x^2-2x-3} dx$

Soln

$$\int \frac{4x-16}{x^2-2x-3} = \frac{4x-16}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3)+B(x+1)}{(x+1)(x-3)}$$

Multiplying all by $(x+1)(x-3)$

$4x-16 = A(x-3) + B(x+1)$

To get the value of A

$f(-1) = 4(-1) - 16 = A(-1-3)$

$-20 = -4A$

$A = 5$

$f(3) = 4(3) - 16 = B(3+1)$

$-4 = 4B$

$B = -1$

Substituting the values of A and B;

$$\frac{4x-16}{x^2-2x-3} = \frac{5}{x+1} - \frac{1}{x-3}$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{5}{x+1} dx - \int \frac{1}{x-3} dx$$

$\int \frac{5}{x+1} dx$; $u = x+1$ $\quad \int \frac{5}{x+1} dx = 5\int \frac{dx}{x+1} = 5\int \frac{du}{u}$

$du/dx = 1$

$dx = du$ $\quad 5\int \frac{du}{u} = 5\ln u = 5\ln(x+1)$

$\int \frac{1}{x-3}dx$; $u = x-3$
$du/dx = 1$
$dx = du$

$\int \frac{1}{x-3}dx = \int \frac{dx}{x-3} = \int \frac{du}{u} = \ln u$

$\ln u = \ln(x-3)$

$\int \frac{4x-16}{x^2-2x-3}dx = 5\ln(x+1) - \ln(x-3) + c$ where c is the constant of integration.

③ $\int$ Simplify $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}dx$

Solution

$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$

Multiplying all by $(x+1)(x-2)(x+3)$,

$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$

To get the value of A;

$f(-1) = 2(-1)^2 - 9(-1) - 35 = A((-1)-2)(-1+3)$

$2+9-35 = -6A$

$A = -24/-6 = 4$

To get the value of B;

$f(2) = 2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$

$= 8 - 18 - 35 = B(15)$

$B = -45/15 = -3$

To get the value of C;

$f(-3) = 2(-3)^2 - 9(3) - 35 = C(-3+1)(-3-2)$

$18 + 27 - 35 = C(10)$

$C = 10/10 = 1$

Substituting the values of A, B and C;

$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$

$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}dx = \int \frac{4}{x+1}dx - \int \frac{3}{x-2}dx + \int \frac{1}{x+3}dx$

$\int \frac{4}{x+1} dx$; $u = x+1$, $du/dx = 1$, $dx = du$

$\int \frac{4}{x+1} dx = 4\int \frac{dx}{x+1} = 4\int \frac{du}{u} = 4\ln u$

$4\ln u = 4\ln(x+1)$

$\int \frac{3}{x-2} dx$; $u = x-2$, $du/dx = 1$, $dx = du$

$\int \frac{3}{x-2} dx = 3\int \frac{dx}{x-2} = 3\int \frac{du}{u}$

$3\ln u = 3\ln(x-2)$

$\int \frac{1}{x+3} dx$; $u = x+1$, $du/dx = 1$, $dx = du$

$\int \frac{1}{x+3} dx = \int \frac{du}{u} = \ln u = \ln(x+3)$

$\therefore \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + c$ where c is the constant of integration.