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MATH 102
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ASYMPTOTICAL EXPANSIONS
18/ENG09/504

1. If $A = 3i + 7j - 2k$, $B = i + 3j + 7k$, $C = 9i - 4j + 6k$ find the angle

(i) A and C

Direction cosine of A

$|A| = \sqrt{3^2 + 7^2 + (-2)^2}$

$= \sqrt{62}$

$l = \frac{3}{\sqrt{62}}, \quad m = \frac{7}{\sqrt{62}}, \quad n = \frac{-2}{\sqrt{62}}$

Direction cosine of C

$|C| = \sqrt{9^2 + (-4)^2 + (6)^2}$

$= \sqrt{133}$

$l = \frac{9}{\sqrt{133}}, \quad m = \frac{-4}{\sqrt{133}}, \quad n = \frac{6}{\sqrt{133}}$

$\cos\theta = ll' + mm' + nn'$

$= \frac{3}{\sqrt{62}} \cdot \frac{9}{\sqrt{133}} + \frac{7}{\sqrt{62}} \cdot \frac{-4}{\sqrt{133}} + \frac{-2}{\sqrt{62}} \cdot \frac{6}{\sqrt{133}}$

$\cos\theta$ = wrong -0.143

$\cos^{-1}(-0.143)$

$= 98.2°$

(ii) B and C

Direction cosine of B

$|B| = \sqrt{1^2 + 3^2 + 7^2}$

$= \sqrt{59}$

$l = \frac{1}{\sqrt{59}}, \quad m = \frac{3}{\sqrt{59}}, \quad n = \frac{7}{\sqrt{59}}$

Direction cosine of c

$|c| = \sqrt{9^2 + 4^2 + 5^2}$

$= \sqrt{133}$

$l = \frac{9}{\sqrt{133}}, \quad m = \frac{-4}{\sqrt{133}}, \quad n = \frac{5}{\sqrt{133}}$

$\cos\theta = ll' + mm' + nn'$

$= \frac{1}{\sqrt{59}} \cdot \frac{9}{\sqrt{133}} + \frac{3}{\sqrt{59}} \cdot \frac{-4}{\sqrt{133}} + \frac{7}{\sqrt{59}} \cdot \frac{6}{\sqrt{133}}$

$= 0.491$

$\cos^{-1}(0.491) =$

$\cos^{-1}(0.491) = 63.8°$

(iii) Unit vector of $(A+B+C)$

$A = 5i + 9j - 2k$

$B = i + 3j + 9k$

$C = 9i - 4j + 6k$

$A+B+C = 13i + 6j + 11k$

$|A+B+C| = \sqrt{13^2 + 6^2 + 11^2}$

$= \sqrt{326}$

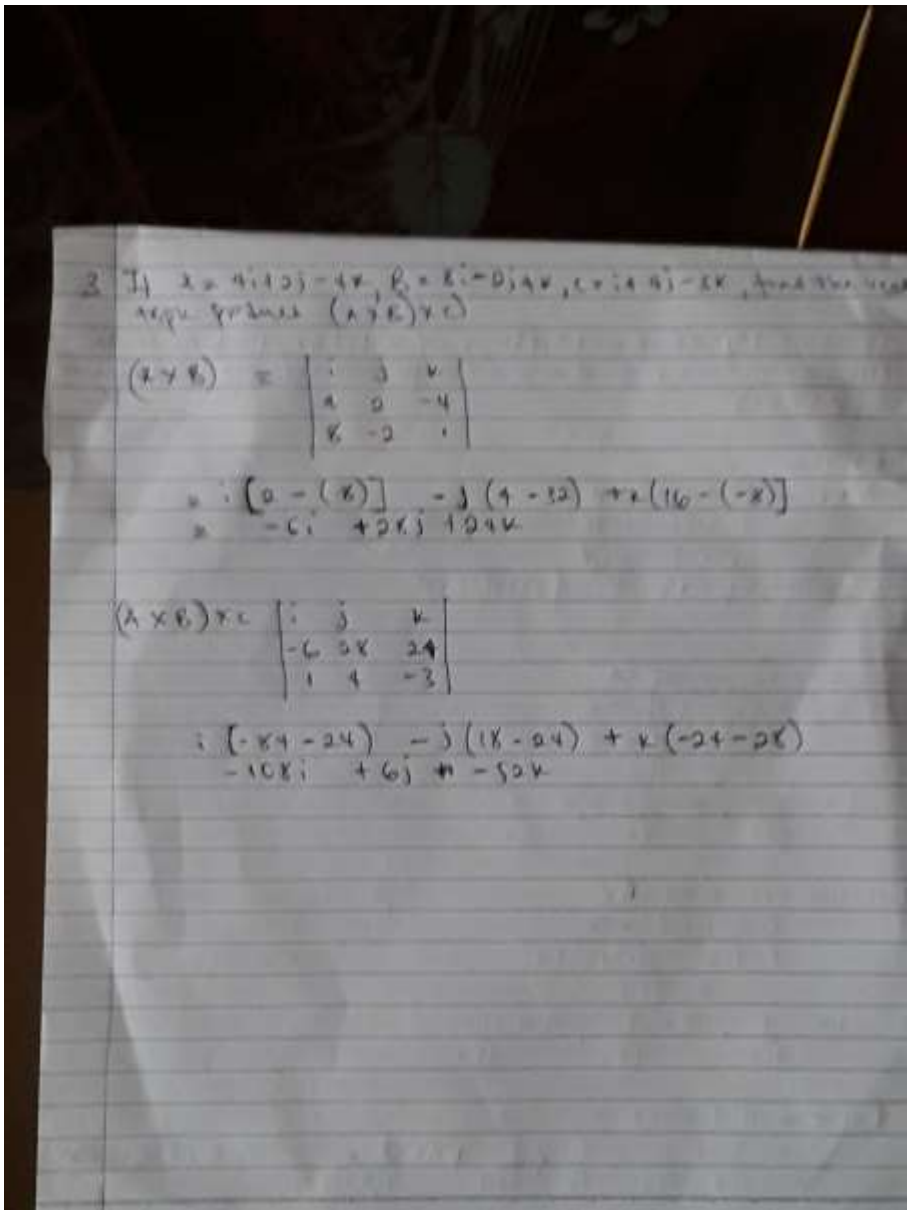
2. A particle moves along a curve, $x = 8t^2$, $y = t^2 - 4t$, $z = t+1$ where t is time find the modulus of acceleration at $t=1$

$\vec{r} = 8t^2 i + (t^2 - 4t) + (t+1)$

$\frac{dr}{dt} = 16t + 2t - 4 + 1$

$\frac{d^2r}{dt^2} = 16 + 2 - 0 + 0$

$\frac{d^2r}{dt^2} = 18$



2 If $A = 4i + 2j - 4k$, $B = 6i - 2j + 4k$, $C = i + 4j - 2k$, find the vector equation $(A \times B) \times C$

$$(A \times B) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 6 & -2 & 4 \end{vmatrix}$$

$$= i[8 - (8)] - j(16 - 32) + k[16 - (-8)]$$

$$= -0i + 24j + 24k$$

$$(A \times B) \times C \begin{vmatrix} i & j & k \\ -0 & 24 & 24 \\ 1 & 4 & -2 \end{vmatrix}$$

$$= i[-48 - 24] - j[0 - 24] + k[0 - 24]$$

$$-120i + 24j + -24k$$