MBBS

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# Integration by Partial fraction

(i) $\int \frac{11-3x}{x^2+2x-3}\,dx$

Solution

$$\frac{B}{(x-1)} - \frac{A}{(x+3)} = \frac{11-3x}{x^2+2x-3}$$

$$\frac{B(x+3) - A(x-1)}{\cancel{(x-1)(x+3)}} = \frac{11-3x}{\cancel{x^2+2x-3}}$$ (∴ since the denominators are the same)

$$Bx + 3B - Ax + A = 11 - 3x$$

$$(B-A)x + (3B+A) = 11 - 3x$$

$$B - A = -3 \quad -- (1) \times 3$$

$$3B + A = 11 \quad -- (2) \times 1$$

$$3B - 3A = -9 \quad -- (3)$$

$$(-)\ 3B + A = 11 \quad -- (4)$$

$$-4A = -20$$

$$\therefore A = 5$$

$$B = -3 + A$$

$$B = -3 + 5$$

$$\therefore B = 2$$

$$\int \frac{2}{(x-1)}\,dx - \int \frac{5}{(x+3)} = \int \frac{11-3x}{x^2+2x-3}$$

| | |
|---|---|
| $u = x - 1$ | $u = x + 3$ |
| $du/dx = 1$ | $du/dx = 1$ |

Continuation of no. 1

$dx = du$

$$\int \frac{2}{(x-1)}\,du - \int \frac{5}{(x+3)}\,du = \int \frac{11-3x}{x^2+2x-3}\,dx$$

$$\therefore \int \frac{11-3x}{x^2+2x-3}\,dx = 2\ln(x-1) - 5\ln(x+3) + C$$

(2) $$\int \frac{4x-16}{x^2-2x-3}\,dx = \frac{A}{(x-3)} - \frac{B}{(x+1)}$$

Solution

$$\frac{A(x+1) - B(x-3)}{\cancel{(x-3)(x+1)}} = \frac{4x-16}{\cancel{x^2-2x-3}}$$ (∴ Since the denominators are equal.)

$$A(x+1) - B(x-3) = 4x - 16$$

When $f(-1)$

$$A(-1+1) - B(-1-3) = 4(-1) - 16$$

$$0 - B(-4) = -4 - 16$$

$$\frac{\cancel{4}B}{\cancel{4}} = \frac{-20}{4}$$

$$\therefore B = -5$$

When $f(3)$

$$A(3+1) - B(3-3) = 4(3) - 16$$

$$4A - 0 = 12 - 16$$

$$\frac{\cancel{4}A}{\cancel{4}} = \frac{-4}{4}$$

$$\therefore A = -1$$

$$\int \frac{4x-16}{x^2-2x-3}\,dx = -\int \frac{1}{(x-3)}\,dx + \int \frac{5}{(x+1)}\,dx$$

Continuation of no. 2.

$$\int \frac{4x-16}{x^2-2x-3} = -\int \frac{1}{(x-3)}\,dx + \int \frac{5}{(x+1)}\,dx$$

$u = x-3$ $\qquad$ $u = x+1$

$du/dx = 1$ $\qquad$ $\frac{du}{dx} = 1$

$\therefore dx = du$

$$\int \frac{4x-16}{x^2-2x-3} = -\int \frac{1}{u}\cdot du + \int \frac{5}{u}\,du$$

$$\int \frac{4x-16}{x^2-2x-3} = -\ln u + 5\ln u + C$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = -\ln(x-3) + 5\ln(x+1) + C_{\text{a}}.$$

OR

$$\int \frac{4x-16}{x^2-2x-3} \qquad 5\ln(x+1) - \ln(x-3) + C_{\text{a}}.$$

(3.) $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$

Solution.

$$\frac{A}{x+1} - \frac{B}{x-2} - \frac{C}{x+3} = \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$$

[$\therefore$ Since the denominators are equal

$A(x-2)(x+3) - B(x+1)(x+3) - C(x+1)(x-2) = 2x^2-9x-35$

When $f(2)$

$0 - B(2+1)(2+3) - 0 = 2(2)^2 - 9(2) - 35$

$-15B = -45$

$\therefore B = \frac{-45}{-15} \Rightarrow 3_{\text{a}}.$

When $f(-1)$

continuation of no. 3.

$A(-1-2)(-1+3) - 0 - 0 = 2(-1)^2 - 9(-1) - 35$

$\frac{-\cancel{6}A}{-\cancel{6}} = \frac{-24}{-6}$

$\therefore A = 4$

When $f(-3)$

$0 - 0 - C(-3+1)(-3-2) = 2(-3)^2 - 9(-3) - 35$

$-10C = 10$

$\therefore C = -1$

$\int \frac{4}{(x+1)} dx - \int \frac{3}{(x-2)} dx + \int \frac{1}{(x+3)} dx = \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$

$u = x+1$ $\quad$ $u = x-2$ $\quad$ $u = x+3$

$du/dx = 1$ $\quad$ $du/dx = 1$ $\quad$ $\frac{du}{dx} = 1$

$\therefore dx = du$

$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{u} du - \int \frac{3}{u} du + \int \frac{dx}{u} du$ $\quad$ $\frac{du}{dx} = 1$ $\quad$ $dx = du$

$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx = 4\ln u - 3\ln u + \ln u + C$

$\therefore \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$