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 Department: Medicine And Surgery
 Course Code: MAT 104
 Course Title: General Mathematics III — Calculus.

Evaluate the following:

- 1) $\int \frac{11-3x}{x^2+2x-3} dx$
- 2) $\int \frac{4x-16}{x^2-2x+3} dx$
- 3) $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$

NO 1

$$\int \frac{11-3x}{x^2+2x-3} dx$$

For x^2+2x-3 : $x^2+3x-x-3 = x(x+3)-1(x-1) = (x-1)(x+3)$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

Multiply through by $(x-1)(x+3)$.

$$11-3x = A(x+3) + B(x-1)$$

At $x = -3$

$$20 = -4B$$

$$B = -5$$

At $x = 1$

$$8 = 4A$$

$$A = 2$$

Hence: $\int \frac{11-3x}{x^2+2x-3} dx = \int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx$

$$\int \frac{11-3x}{x^2+2x-3} dx = 2 \ln|x-1| - 5 \ln|x+3|$$

$$2) \int \frac{4x-16}{x^2-2x-3} dx$$

For x^2-2x-3 : $x^2-3x+x-3 = x(x-3) + (x-3) = (x+1)(x-3)$

$$\frac{4x-16}{x^2-2x-3} = \frac{4x-16}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x-3)(x+1)}$$

Multiply through by $(x-3)(x+1)$

$$4x-16 = A(x-3) + B(x+1)$$

At $x = 3$

$$-4 = 4B$$

$$B = -1$$

At $x = -1$

$$-20 = -4A$$

$$A = 5$$

Hence: $\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{5}{x+1} dx - \int \frac{dx}{x-3}$

$$\int \frac{4x-16}{x^2-2x-3} dx = 5 \ln|x+1| - \ln|x-3| + C$$

NO3

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx = \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Multiply

through by $(x+1)(x-2)(x+3)$

$$2x^2-9x-35$$

$$= A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$2x^2-9x-35$$

$$= A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)$$

$$2x^2-9x-35$$

$$= Ax^2 + Bx^2 + Cx^2 + Ax + 4Bx - Cx - 6A + 3B - 2C$$

$$2x^2-9x-35$$

$$= (A+B+C)x^2 + (A+4B-C)x - (6A-3B+2C)$$

Equate the coefficients of x^2 : $A+B+C = 2$ — ①

Equate the coefficient of x : $A+4B-C = -9$ — ②

Equate the coefficient of x^0 : $6A - 3B + 2C = 35$ — (3)

From 1, 2, 3, eliminate C: $2A + 5B = -7$ — (4)

$8A + 5B = 17$ — (5)

$6A = 24$

$A = 4$

Put 4 for A in equ (4)

$8(4) + 5B = 17$

$5B = -15$

$B = -3$

Put 4 for A, -3 for B in equ (3)

$6(4) - 3(-3) + 2C = 35$

$2C = 2$

$C = 1$

Hence: $\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx - \int \frac{3}{x-2} dx + \int \frac{dx}{x+3}$

$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3|$