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1.) $\int \frac{11-3x}{x^2+2x-3} dx$

Solution

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

Multiply through by $(x-1)(x+3)$

$11-3x = A(x+3)+B(x-1)$

$11-3x = Ax+3A+Bx-B$

$11-3x = (A+B)x + (3A-B)$

$A+B = -3 \quad --(1) \quad \times 3$

$3A-B = 11 \quad --(2) \quad \times 1$

$3A+3B = -9 \quad --(3)$

$3A-B = 11 \quad --(4)$

$$\frac{4B}{4} = -\frac{20}{4}$$

$B = -5$

Put $B = -5$ in (1)

$A+(-5) = -3$

$A-5 = -3$

$A = -3+5$

$A = 2$

$\therefore A = 2$ and $B = -5$

$$\int \frac{2dx}{x-1} + \int \frac{-5}{x+3}dx$$

Let $u = x-1$
$\frac{du}{dx} = 1$
$du = dx$
$\therefore = 2\int \frac{du}{u}$

Let $u = x+3$
$\frac{du}{dx} = 1$
$du = dx$
$\therefore -5\int \frac{du}{u}$

$= 2\ln u + (-5\ln u)$
$= 2\ln(x-1) - 5\ln(x+3) + C$

2) $\int \frac{4x-16}{x^2-2x-3}dx$

Solution

$\frac{4x-16}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$

$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3)+B(x+1)}{(x+1)(x-3)}$

Multiply through by $(x+1)(x-3)$.

$4x-16 = A(x-3)+B(x+1)$
$4x-16 = Ax-3A+Bx+B$
$4x-16 = (A+B)x-(3A-B)$
$A+B = 4$ -- (1) $\times 3$
$-3A+B = -16$ -- (2) $\times 1$

$3A+3B = 12$ -- (3)
$+ \quad -3A+B = -16$ -- (4)
$\frac{4B}{4} = \frac{-4}{4}$
$B = -1$

Put $B=-1$ in (1)

$A+(-1)=4$

$A-1=4$

$A=4+1$

$A=5$

$\therefore A=5$ and $B=-1$

$$\int \frac{5dx}{(x+1)} + \int \frac{-1}{(x-3)}dx$$

Let $u=x+1$

$\frac{du}{dx}=1$

$du=dx$

$\therefore = 5\int \frac{du}{u}$

Let $u=x-3$

$\frac{du}{dx}=1$

$du=dx$

$\therefore = -1\int \frac{du}{u}$

$= 5\ln u + (-1\ln u)$

$= 5\ln(x+1) - \ln(x-3) + c$

3) $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}dx$

Solution

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x+2)}{(x+1)(x-2)(x+3)}$$

Multiply through by $(x+1)(x-2)(x+3)$.

$2x^2-9x-35 = A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x-2)$

$2x^2-9x-35 = Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C$

$2x^2-9x-35 = (A+B+C)x^2+(A+4B-C)x+(-6A+3B-2C)$

$A+B+C=2$ --①

$A+4B-C=-9$ --②

$-6A+3B-2C=-35$ --③

From ①

$A=2-B-C$ --④

Put ④ in ②

$2-B-C+4B-C=-9$

$-B+4B-C-C=-9-2$

$3B-2C=-11$

$\frac{3B}{3}=\frac{-11+2C}{3}$

$B=\frac{-11+2C}{3}$ --⑤

Also Put ④ in ③

$-6(2-B-C)+3B-2C=-35$

$-12+6B+6C+3B-2C=-35$

$6B+3B+6C-2C=-35+12$

$9B+4C=-23$ --⑥

Put ⑤ in ⑥

$3\cancel{9}\left(\frac{-11+2C}{3\cancel{1}}\right)+4C=-23$

$3(-11+2C)+4C=-23$

$-33+6C+4C=-23$

$10C=-23+33$

$10C=10$

$C=\frac{10}{10}$

$C=1$

Put $C=1$ in ⑤

$B=\frac{-11+2(1)}{3}$

$$B = \frac{-11+2}{3}$$

$$B = \frac{-9}{3}$$

$$B = -3$$

Put $B = -3$ and $C = 1$ in (4)

$$A = 2 - (-3) - 1$$

$$A = 2 + 3 - 1$$

$$A = 5 - 1$$

$$A = 4$$

$\therefore A = 4,\ B = -3$ and $C = 1$

$$\therefore \int \frac{4}{(x+1)}dx + \int \frac{-3}{(x-2)}dx + \int \frac{1}{(x+3)}dx$$

Let $u = x+1$ | Let $u = x-2$ | Let $u = x+3$

$\frac{du}{dx} = 1$ | $\frac{du}{dx} = 1$ | $\frac{du}{dx} = 1$

$du = dx$ | $du = dx$ | $du = dx$

$\therefore = 4\int \frac{du}{u}$ | $\therefore = -3\int \frac{du}{u}$ | $\therefore = 1\int \frac{du}{u}$

$= 4 \ln u$ | $-3 \ln u$ | $\ln u$

$\therefore = 4\ln(x+1) + (-3\ln(x-2)) + \ln(x+3)$

$= 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + c$