

Question 1

1)

$$\int \frac{11-3x}{x^2+2x-3} dx$$

Solve

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$11-3x = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$11-3x = A(x+3) + B(x-1)$$

We know from

$$(x-1)(x+3), x=1 \text{ OR } -3$$

Find A

$$\text{Let } x=1$$

$$11-3(1) = A(4) + B(1-1)$$

$$\frac{8}{4} = \frac{4A}{4} \quad A = 2$$

Find B

$$\text{Let } x=-3$$

$$11-3(-3) = A(-3+3) + B(-3-1)$$

$$11+9 = 0 + B(-4)$$

$$\frac{20}{-4} = \frac{-4B}{-4}$$

$$B = -5$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$$

$$\int \frac{11-3x}{(x-1)(x+3)} dx = \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$$

Take

$$\int \frac{2}{x-1} dx$$

$$u = x-1$$

$$du = 1$$

$$dx$$

$$dx = du$$

$$\int \frac{11-3x}{(x-1)(x+3)} dx = \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$$

Consider

$$\int \frac{2}{x-1} dx = \frac{2}{x-1} \int du$$

$$u = x-1$$

$$du = 1$$

$$dx = du$$

$$= 2 \int \frac{1}{u} du$$

$$= 2 \ln u = 2 \ln(x-1)$$

Consider

$$\int \frac{5}{x+3} dx \quad u = x+3$$

$$\frac{du}{dx} = 1$$

$$\delta x = \delta u$$

$$\int \frac{5 \delta x}{(x+3)} = \int \frac{5 \cdot \delta u}{u}$$

$$= 5 \int \frac{1}{u} \cdot \delta u$$

$$= 5 \ln u = 5 \ln(x+3)$$

$$\int \frac{11-3x}{(x^2+2x-3)} \delta x = 2 \ln(x-1) - 5 \ln(x+3)$$

$$= 2 \ln(x-1) - 5 \ln(x+3) + C \quad // \text{ANS}$$

2) Question 2

$$\int \frac{4x-16}{(x^2-2x-3)} \delta x$$

Solu

$$\frac{4x-16}{(x^2-2x-3)} = \frac{4x-16}{(x+1)(x-3)}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$$
$$A(x-3) + B(x+1) = (x+1)(x-3)$$

$$4x-16 = A(x-3) + B(x+1)$$

We know From

$$(x+1)(x-3), x = -1 \text{ OR } +3$$

Find A

$$\text{Let } x = -1$$

$$4(-1)-16 = A(-1-3) + B(-1+1)$$

$$-4-16 = A(-4) + B(0)$$

$$-20 = -4A$$

$$A=5$$

Find B

$$\text{Let } x = 3$$

$$4(3)-16 = A(3-3) + B(3+1)$$

$$12-16 = 0 + 4B$$

$$-4 = 4B$$

$$B = -1$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{5}{(x+1)} - \frac{1}{(x-3)}$$

$$\int \frac{4x-16}{(x+1)(x-3)} \delta x = \int \frac{5 \delta x}{(x+1)} - \int \frac{1 \delta x}{(x-3)}$$

$$\int \frac{4x-16}{(x+1)(x-3)} \delta x = 5 \ln(x+1) - \ln(x-3) + C \quad // \text{ANS}$$

$$\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)} \delta x$$

Solu

$$= \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = (x+1)(x-2)(x+3)$$

$$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$\text{Let } x = -1$$

$$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1-2)$$

$$= \ln(x) + \ln(x+1) + \ln(x+2) - 3 \ln(x) + C // \text{Ans}$$

$$-24 = A(-3)(2)$$

$$-24 = -6A$$

$$A = 4$$

$$\text{Let } x = 2$$

$$2(2)^2 - 9(2) + 35 = B(2+1)(2+3)$$

$$8 - 18 - 35 = B(3)(5)$$

$$-45 = 15B$$

$$B = -3$$

$$\text{Let } x = -8$$

$$2(-8)^2 - 9(-8) - 35 = C(-8+1)(-8-2)$$

$$18 + 27 - 35 = C(-7)(-10)$$

$$10 = C(70)$$

$$C = 1$$

$$\frac{2x^2 - 9x + 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$$

$$+ \frac{1}{x+3}$$

$$(x+3)$$

$$\int \frac{2x^2 - 9x + 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx - \int \frac{3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$(x+1)(x-2)(x+3)$$

$$4 \int \frac{1}{x+1} dx - 3 \int \frac{1}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\int \frac{1}{x+3} dx$$

$$\int \frac{1}{x-2} dx$$