

$$1. \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{A}{x-1} + \frac{B}{x+3} dx$$

Solve for:

$$\frac{11-3x}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3}$$

Multiply through by $(x-1)(x+3)$

$$11-3x = A(x+3) + B(x-1)$$

when $x=1$

$$11-3(1) = A(1+3) + B(1-1)$$

$$8 = 4A$$

$$A = 8/4 = 2$$

when $x=-3$

$$11-3(-3) = A(-3+3) + B(-3-1)$$

$$11+9 = -4B$$

$$\frac{20}{-4} = \frac{-4B}{-4}$$

$$B = -5$$

$\therefore A=2, B=-5$

$$\int \frac{11-3x}{(x-1)(x+3)} = \int \frac{2}{x-1} + \frac{-5}{x+3} dx$$

$$= \int \frac{2 dx}{x-1} + \int \frac{-5}{x+3} dx$$

when $u = x-1$ and $u = x+3$

$$du = dx$$

$$du = dx$$

$$= 2 \int \frac{1}{u} du + -5 \int \frac{1}{u} du$$

$$= 2 \ln u + (-5) \ln u + C$$

$$= 2 \ln(x-1) - 5 \ln(x+3) + C$$

$$2. \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3} dx$$

Solve for:

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

Multiply through by $(x+1)(x-2)(x+3)$

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

when $x = 2$

$$2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$$

$$8 - 18 - 35 = B(3)(5)$$

$$\frac{-45}{15} = 3B$$

$$B = -3$$

$$B = -3$$

when $x = -1$

$$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3)$$

$$2 + 9 - 35 = A(-3)(2)$$

$$-24 = -6A$$

$$A = 4$$

when $x = -3$

$$2(-3)^2 - 9(-3) - 35 = C(-3+1)(-3-2)$$

$$18 + 27 - 35 = C(-2)(-5)$$

$$45 - 35 = 10C$$

$$\frac{10}{10} = \frac{10C}{10}$$

$$C = 1$$

$$\therefore A = 4, B = -3, C = 1$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} + \frac{-3}{x-2} + \frac{1}{x+3} dx$$

when $u = x+1$, $u = x-2$, $u = x+3$

$$du = dx \quad du = dx \quad du = dx$$

$$= 4 \int \frac{1}{x+1} dx - 3 \int \frac{1}{x-2} dx + \int \frac{1}{x+3} dx$$

$$= 4 \ln u - 3 \ln u + \ln u + c$$

$$3. \int \frac{4x-16}{x^2-2x-3} dx = \int \frac{A}{(x+1)} + \frac{B}{(x-3)} dx$$

solve for:

$$\frac{4x-16}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$$

Multiply through $(x+1)(x-3)$

$$4x - 16 = A(x-3) + B(x+1)$$

when $x = 3$

$$4(3) - 16 = A(3-3) + B(3+1)$$

$$12 - 16 = 4B$$

$$4B = -4$$

$$B = -1$$

when $x = -1$

$$4(-1) - 16 = A(-1-3) + B(-1+1)$$

$$-4 - 16 = -4A$$

$$-20 = -4A$$

$$A = 5$$

$$\therefore A = 5, B = -1$$

$$\int \frac{4x-16}{x^2-2x-3} = \int \frac{5}{x+1} + \frac{-1}{x-3} dx$$

$$= 5 \int \frac{1}{x+1} dx - \int \frac{1}{x-3} dx$$

$$= 5 \ln|x+1| - \ln|x-3| + C$$