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DEPARTMENT: MEDICINE AND SURGERY

ASSIGNMENT

$$(1) \int \frac{11-3x}{x^2+2x-3} dx$$

Solution

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$11-3x = A(x+3) + B(x-1)$$

$$11-3x = Ax + 3A + Bx - B$$

$$11-3x = Ax + Bx + 3A - B$$

$$11-3x = (3A - B) + (A+B)x$$

By comparing the ~~coeff~~ coefficients,

$$3A - B = 11$$

$$A + B = -3$$

$$4A = 8$$

$$A = 2$$

putting $A = 2$ into equation (ii)

$$2 + B = -3$$

$$B = -3 - 2$$

$$B = -5$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{2}{(x-1)} - \frac{5}{(x+3)}$$

$$\int \frac{11-3x}{x^2+2x-3} dx = \int \left(\frac{2}{x-1} - \frac{5}{x+3} \right) dx = \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$$

$$= 2 \int \frac{1}{x-1} dx - 5 \int \frac{1}{x+3} dx$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = 2 \ln|x-1| - 5 \ln|x+3| + C$$

$$(2) \int \frac{4x-16}{x^2-2x-3} dx$$

Solution

$$\frac{4x-16}{x^2-2x-3} = \frac{4x-16}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

$$4x-16 = A(x-3) + B(x+1)$$

$$4x-16 = Ax-3A+Bx+B$$

$$4x-16 = Ax+Bx-3A+B$$

$$4x-16 = (A+B)x-3A+B$$

By comparing the coefficients,

$$A+B = 4 \quad \text{--- equation (i) } \times -1$$

$$-3A+B = -16 \quad \text{--- equation (ii) } \times 1$$

$$-A-B = -4$$

$$-3A+B = -16$$

$$-4A = -20$$

$$A = 5$$

Putting $A=5$ into equation (i)

$$A+B = 4$$

$$B = 4-A$$

$$B = 4-5$$

$$B = -1$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{5}{x+1} - \frac{1}{x-3}$$

$$\int \frac{4x-16}{(x+1)(x-3)} dx = \int \left(\frac{5}{x+1} - \frac{1}{x-3} \right) dx$$

$$= \int \frac{5}{x+1} dx - \int \frac{1}{x-3} dx$$

$$= 5 \int \frac{1}{x+1} - \int \frac{1}{x-3}$$

$$= 5 \ln(x+1) - \ln(x-3) + C$$

$$\therefore \int \frac{4x-16}{(x+1)(x-3)} dx = 5 \ln(x+1) - \ln(x-3) + C$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} dx = 5 \ln(x+1) - \ln(x-3) + C$$

$$(3) \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$$

Solution.

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A[(x-2)(x+3)]}{(x+1)(x-2)(x+3)} + \frac{B[(x+1)(x+3)]}{(x+1)(x-2)(x+3)} + \frac{C[(x+1)(x-2)]}{(x+1)(x-2)(x+3)}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2-9x-35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + (x^2 - (x-2)C)$$

$$\begin{aligned} 2x^2-9x-35 &= Ax^2 + Bx^2 + (x^2 + Ax + 4Bx - Cx - 6A + 3B - 2C) \\ &= (A+B+C)x^2 + (A+4B-C)x - 6A+3B-2C \end{aligned}$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x - 6A + 3B - 2C$$

By comparing the coefficients,

$$A + B + C = 2 \text{ ----- equation (i)}$$

$$A + 4B - C = -9 \text{ ----- equation (ii)}$$

$$-6A + 3B - 2C = -35 \text{ ----- equation (iii)}$$

By comparing (i) and (ii)

$$A + B + C = 2$$

$$A + 4B - C = -9$$

$$2A + 5B = -7 \text{ ----- equation (iv)}$$

By comparing (i) and (iii)

$$A + B + C = 2 \text{ ----- equation (i) } \times 2$$

$$-6A + 3B - 2C = -35 \text{ ----- equation (iii) } \times 1$$

$$2A + 2B + 2C = 4$$

$$-6A + 3B - 2C = -35$$

$$-4A + 5B = -31 \text{ ----- equation (v)}$$

Solve equations (iv) and (v) simultaneously.

$$2A + 5B = -7$$

$$-4A + 5B = -31$$

$$2A + 5B = -7 \text{ ----- equation (iv) } \times 1$$

$$4A - 5B = -31 \text{ ----- equation (v) } \times -1$$

$$6A = 24$$

$$A = 4$$

Putting $A=4$ into equation (iv)

$$2(4) + 5B = -7$$

$$5B = -7 - 8$$

$$5B = -15$$

$$B = -3$$

Putting $A=4$, $B=-3$ into equation (i)

$$4 - 3 + C = 2$$

$$C = 2 - 1$$

$$C = 1$$

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$$\therefore A = 4, b = -3 \text{ and } C = 1$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \left(\frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)} \right) dx$$
$$= 4 \int \frac{1}{(x+1)} dx - 3 \int \frac{1}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3) + C$$